Workshop on Global and Microlocal Analysis

28-29 November 2023

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA
DEPARTMENT OF MATHEMATICS

Invited Speakers

- Marco Cappiello (University of Torino)
- Sandro Coriasco (University of Torino)
- Stefan Fürdös (University of Vienna)
- Michael Hitrik (UCLA Los Angeles)
- Howard Jacobowitz (Rutgers University)
- Max R. Jahnke (Universität zu Köln)
- David Jornet (Universitat Politècnica de València)
- Federico Riccardi (Politecnico of Torino)
- Gerhard Schindl (University of Vienna)
- Jörg Seiler (University of Torino)

Organizers

- Gregorio Chinni (Università di Bologna)
- Marco Mughetti (Università di Bologna)

Program

November 28

2:00-2:15 pm Opening Remarks

2:15-3:00 pm I Talk Howard Jacobowitz

"CR Geometry and Analysis"

3:00-3:45 pm II Talk Marco Cappiello

"The Cauchy problem for
p-evolution equations in Gevrey Spaces"

3:45-4:30 pm III Talk Stefan Fürdös

"Ellipticity and the problem of iterates
in Denjoy-Carleman classes"

4:30-5:00 pm Coffee Break

5:00-5:45 pm IV Talk Jörg Seiler

"Parametric pseudodifferential operators
with point-singularity in the covariable"

November 28

5:45-6:30 pm V Talk Sandro Coriasco

"The Weyl asymptotic of elliptic operators on a class of noncompact manifolds"

6:30-7:15 pm VI Talk Michael Hitrik

"Analytic hypoellipticity in the chiral model of twisted bilayer graphene"

November 29

9:00-9:45 am VII Talk Federico Riccardi

"Recent results on the norm of localization operators"

9:45-10:30 am VIII Talk Max R. Jahnke

"The cohomology of left-invariant involutive structures on compact Lie groups"

10:30-11:00 am Break

11:00-11:45 am IX Talk Gerhard Schindl

"Ultradifferentiable classes of entire functions"

11:45-12:30 am X Talk David Jornet

"Mean-dispersion principles and the Wigner transform"

List of Abstracts

The Cauchy problem for p-evolution equations in Gevrey spaces

Marco Cappiello
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We study the Cauchy problem

$$\begin{cases} P(t,x,D_t,D_x)u(t,x) = f(t,x) \\ u(0,x) = g(x) \end{cases}, \qquad (t,x) \in [0,T] \times \mathbb{R}, \tag{0.1}$$

for p-evolution operators of the form

$$P(t,x,D_t,D_x) = D_t + a_p(t)D_x^p + \sum_{j=1}^{p-1} a_j(t,x)D_x^j, \qquad (t,x) \in [0,T] \times \mathbb{R},$$

where $a_p \in C([0,T],\mathbb{R})$ and $a_j \in C([0,T],C^\infty(\mathbb{R};\mathbb{C}))$, $j = 0,\dots,p-1$, in the Gevrey functional setting. When the coefficients $a_j(t,x)$, $j = 0,\dots,p-1$, of the lower order terms are complex-valued, it is possible to obtain well-posedness results in Gevrey spaces under suitable decay assumptions on a_j for $|x| \to \infty$. In the first part of the talk, we present a well-posedness result for 3-evolution equations obtained in [1]. In the second part we discuss necessary conditions for Gevrey well-posedness in the case of p-evolution equations for an arbitrary positive integer p, see [2]. The results presented in the talk are obtained in collaboration with Alexandre Arias Junior (Università di Torino) and Alessia Ascanelli (Università di Ferrara).

References

1 A. Arias Junior, A. Ascanelli, M. Cappiello, *Gevrey well-posedness for* 3*-evolution equatons with variable coefficients*, 2022. To appear in Ann. Scuola Norm. Sup. Pisa Cl. Sci. DOI: 10.2422/2036-2145.202202_011, https://arxiv.org/abs/2106.09511

2 A. Arias Junior, A. Ascanelli, M. Cappiello, *On the Cauchy problem for p-evolution equations with variable coefficients: a necessary condition for Gevrey well-posedness.* Preprint (2023), https://arxiv.org/abs/2309.05571

The Weyl asymptotics of elliptic operators on a class of noncompact manifolds

Sandro Coriasco
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We illustrate the asymptotic behaviour of the eigenvalue counting function for self-adjoint, positive, elliptic linear operators, defined through classical weighted symbols of order $(1,1)$, on an asymptotically Euclidean manifold X. We first prove a two term Weyl formula, improving previously known remainder estimates. Subsequently, we show that, under a geometric assumption on the Hamiltonian flow at infinity, there is a refined Weyl asymptotics with three terms. This is joint work with Moritz Doll.

Ellipticity and the problem of iterates in Denjoy-Carleman classes

Stefan Fürdös
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In 1978 Metivier showed that a differential operator with real-analytic coefficients is elliptic if and only if any non-analytic Gevrey vector is a Gevrey function of the same order. In this talk we generalize Metivier's Theorem to Denjoy-Carleman classes given by weight sequences. In particular we show that if $\mathcal{E}^{\{\mathbf{M}\}}$ is a Denjoy-Carleman class such that the associated Borel map is surjective, then there is a vector u of class $\{\mathbf{M}\}$ for any non-elliptic differential operator with real-analytic coefficients, which is not an element of $\mathcal{E}^{\{\mathbf{M}\}}$. This is joint work with Gerhard Schindl.

Analytic hypoellipticity in the chiral model of twisted bilayer graphene

Michael Hitrik
UCLA Los Angeles
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Magic angles are a topic of current interest in condensed matter physics: when two sheets of graphene are twisted by those angles the resulting material is superconducting. In this talk, we shall discuss a simple operator describing the chiral limit of twisted bilayer graphene, whose spectral properties are thought to determine which angles are magical. It comes from a 2019 PR Letter by Tarnopolsky–Kruchkov–Vishwanath. By adapting analytic hypoellipticity results of Kashiwara, Trepreau, Sjöstrand, and Himonas, we show that the corresponding eigenfunctions decay exponentially in suitable geometrically determined regions, as the angle of twisting decreases. This is joint work with Maciej Zworski.

CR Geometry and Analysis

Howard Jacobowitz
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IThis talk will be an overview of some relations between the geometry and analysis of CR structures in three dimensions. The subject began with Poincare's observation that in 2-dimensional complex space there are more real hypersurfaces through a given point than there are local biholomorphisms leaving that point fixed. Elie Cartan then computed the geometric invariants that solve the local equivalency problem. Hans Lewy studied Cartan's work, in particular the partial differential operator on the hyperquatric and its relation to the underlying geometry. This led to his famous counterexample - a linear partial differential equation with no solution. The talk will conclude with the realization problem and a false analogy.

The cohomology of left-invariant involutive structures on compact Lie groups

Max R. Jahnke
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It is well known that the De Rham cohomology of a compact Lie group is isomorphic to the Chevalley-Eilenberg complex. While the former is a topological invariant of the Lie group, the latter can be computed by using simple linear algebra methods. In this talk, we discuss how to obtain an injective homomorphism between the cohomology spaces associated with left-invariant involutive structures and the cohomology of a generalized Chevalley-Eilenberg complex. We discuss some cases in which the homomorphism is surjective, such as the Dolbeault cohomology and certain elliptic and CR structures. The results provide new insights regarding the general theory of involutive structures as, for example, they reveal algebraic obstructions for solvability for the associated differential complexes.

Mean-dispersion principles and the Wigner transform

David Jornet
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Given a function $f \in L^2(\mathbb{R})$, we consider means and variances associated to f and its Fourier transform $\hat{f}$, and explore their relations with the Wigner transform $W(f)$, obtaining, as particular cases, a simple new proof of Shapiro's mean-dispersion principle, as well as a stronger result due to Jaming and Powell. Uncertainty principles for orthonormal sequences in $L^2(\mathbb{R})$ involving linear partial differential operators with polynomial coefficients and the Wigner distribution, or different Cohen class representations, are obtained, and an extension to the case of Riesz bases is studied.

This is a joint work with Chiara Boiti (Università degli Studi di Ferrara) and Alessandro Oliaro (Università di Torino)

Recent results on the norm of localization operators

Federico Riccardi
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Time-frequency localization operators (with Gaussian window) were introduced by Daubechies in 1988. Since then they have been studied intensively, in particular regarding boundedness, compactness, Schatten properties and estimates for the eigenvalues. However, sharp estimates for the norm of these operators are still few. In this talk I will present a classical result by Lieb and two new result that give sharp estimates for the norm of localization operators under the assumption that the weight function belongs to one or more L^p spaces.

Ultradifferentiable classes of entire functions

Gerhard Schindl
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We study classes of ultradifferentiable functions defined in terms of small weight sequences violating standard growth and regularity requirements. First, we show that such classes can be viewed as weighted spaces of entire functions for which the crucial weight is given by the associated weight function of the so-called conjugate weight sequence. Moreover, we generalize results from M. Markin from the so-called small Gevrey-setting to arbitrary convenient families of (small) sequences and show how the corresponding ultradifferentiable function classes can be used to detect boundedness of normal linear operators on Hilbert spaces (associated to an evolution equation problem). Finally, we study the connection between small sequences and the recent notion of dual sequences introduced in the PhD-thesis of Javier Jiménez-Garrido.
This is joint work with David Nicolas Nenning from the University of Vienna.

Parametric pseudodifferential operators with point-singularity in the covariable

Jörg Seiler
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Starting out from a new description of a class of parameter-dependent pseudodifferential operators with finite regularity number due to G. Grubb, we introduce a calculus of parameter-dependent, poly-homogeneous symbols whose homogeneous components have a particular type of point-singularity in the covariable-parameter space. Such symbols admit intrinsically a second kind of expansion which is closely related to the expansion in the Grubb-Seeley calculus and permits to recover the resolvent-trace expansion for elliptic pseudodifferential oerators originally proved by Grubb-Seeley. Another application is the invertibility of parameter-dependent operators of Toeplitz type, i.e., operators acting in subspaces determined by zero-order pseudodifferential idempotents.