Geometry of Information Theory

@ UC3M Madrid, 29 September 2023

09:15 / 09:30 Opening

09:30 / 10:15 Goffredo Chirco

Title: TBA

Abstract: TBA

10:15 / 11:00 Paolo Perrone

Title: Information theory with Markov categories

Abstract: Markov categories are a novel framework to describe and treat problems in probability and information theory. By means of a diagrammatic calculus one has then notions of stochastic dependence and independence, determinism and randomness, and conditioning. Several theorems of probability and statistics can be recast and reproven in this language, for example de Finetti's theorem and the Hewitt-Savage zero-one law.
One can combine this categorical formalism with the traditional quantitative notions of entropy, mutual information, and data processing inequalities. Several quantitative aspects of information theory can be captured by an enriched version of Markov categories, where the spaces of morphisms are equipped with a divergence or even a metric.
For instance, Markov categories give a notion of determinism for sources and channels, and we can define entropy exactly by measuring how far a source or channel is from being deterministic. This recovers Shannon and Rényi entropies, as well as the Gini-Simpson index used in ecology to quantify diversity, and it can be used to give a conceptual definition of generalized entropy.

11:00 / 11:30 Coffe break

11:30 / 12:15 Eliot Tron

Title: Canonical foliations of neural networks: application to robustness

Abstract: Deep learning models are known to be vulnerable to adversarial attacks. Adversarial learning is therefore becoming a crucial task. We propose a new vision on neural network robustness using Riemannian geometry and foliation theory. The idea is illustrated by creating a new adversarial attack that takes into account the curvature of the data space. This new adversarial attack called the two-step spectral attack is a piece-wise linear approximation of a geodesic in the data space. The data space is treated as a (degenerate) Riemannian manifold equipped with the pullback of the Fisher Information Metric (FIM) of the neural network. In most cases, this metric is only semi-definite and its kernel becomes a central object to study. A canonical foliation is derived from this kernel. The curvature of transverse leaves gives the appropriate correction to get a two-step approximation of the geodesic and hence a new efficient adversarial attack. The method is first illustrated on a 2D toy example in order to visualize the neural network foliation and the corresponding attacks. Next, experiments on the MNIST dataset with the proposed technique and a state of the art attack presented in Zhao et al. (2019) are reported. The result show that the proposed attack is more efficient at all levels of available budget for the attack (norm of the attack), confirming that the curvature of the transverse neural network FIM foliation plays an important role in the robustness of neural networks

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12:15 / 13:00 Rita Fioresi

Title: Deep learning and information geometry

Abstract: We study metrics in deep learning parameter and data spaces, discovering different and geometrically interesting behavior.

15:00 / 15:45 Paolo Gibilisco

Title: "Higher mathematics from an elementary point of view": how to understand (and prove?) the Petz conjecture for the scalar curvature of BKM metric.

Abstract: In 1994 Denes Petz proposed a conjecture on the behavior of scalar curvature of the Bogoliubov-Kubo-Mori metric. The BKM metric is one of the metrics, on quantum state space, which are called Quantum Fisher Information(s) because of Petz classification theorem. In this talk I'll show that the conjecture can be understood starting from somehow elementary considerations on the curvature of the L^p spheres in the cartesian plane. This understanding produces what seems a reasonable proof strategy for the Petz conjecture itself.

15:45 / 16:30 Lorenz Schwachhöfer

Title: Information Geometry in the classical and quantum setting

Abstract: In this talk, we shall give an outline on developments in classical and quantum Information Geometry. We shall give an introduction to the subject, but also present some recent results in collaboration with F. Ciaglia (Madrid), J. Jost and F. di Nocera (both Leipzig).

16:30 / 17:00 Coffe break

17:00 / 17:45 Alberto Ibort

Title: TBA

Abstract: TBA

17:45 / 18:00 Closing