Combinatorial integrability Minicourse

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The minicourse will be devoted to a description of combinatorial solutions to integrable hierarchies of Kadomtsev–Petviashvili type that arise naturally in enumeration of various topological and algebro-geometric objects. A preliminary layout includes

- *Permutations and their decompositions into products of transpositions* (simple Hurwitz numbers, Okounkov's theorem, Hurwitz formula, Bousquet-Mélou–Schaeffer formula, cut-and-join equation)

- *Symmetric group representations* (diagonilizability of the cut-and-join operator, the group algebra of the symmetric group, Schur polynomial, Jucys–Murphy elements)

- *The semiinfinite Grassmannian and the Kadomtsev–Petviashvili hierarchy* (Plücker embeddings, semiinfinite planes in the space of Laurent series, Orlov–Shcherbin family of solutions)

- *Ramified coverings of the* 2*-sphere* (coverings and ramified coverings, Hurwitz numbers and ramified coverings, Caley formula, genus expansion)

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