The Hitchin fibration as a completely integrable system

After recalling some well-known facts about period matrices of Riemann surfaces, I will show how abelianization, in the form of the Beauville-Narasimhan-Ramanan correspondence, produces Higgs bundles associated with spectral curves. The moduli space of Higgs bundles on a given Riemann surface is a partial compactification of the cotangent bundle of the moduli space of vector bundles, and is therefore endowed with a symplectic structure. The relative Picard scheme of the spectral family acts on this moduli space, thus giving the completely integrable structure. I will quickly discuss the structure of the singular fibres of the Hitchin map. For simplicity only the case of $GL_n$-Higgs bundles will be treated.