

CA21109 COST Action CaLISTA

## CaLISTA Kick-off Meeting

June 5 - 9 , 2023

# ABSTRACTS

<https://events.unibo.it/kom-calista2023>

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# Abstracts

JESUS ANGULO LOPEZ

*Mines Paris – PSL, France*

## **A morphological representation theory of equivariant CNNs on homogenous spaces.**

We are interested in this talk on a nonlinear theory of equivariant convolutional neural networks (CNNs) on homogenous spaces under the action of a group. Many groups of image transforms fit this framework.

JESUS BALADO-FRIAS

*Universidade de Vigo, Spain*

## **Structuring elements of mathematical morphology to guide semantic point cloud segmentation.**

Geometric feature search is a very active field in semantic segmentation and classification problems with point clouds. Most artificial intelligence algorithms assume very simple searches: number of nearest neighbors, search spheres, cubes, or cylinders. In this work we propose the use of structuring elements to obtain complex geometry to improve point cloud semantic segmentation. We define the use of a geometric form (seven points aligned with orthogonal vectors) that takes advantage of the direction of the points in the XYZ space. Twelve local geometric features (linearity, planarity, scattering, curvature, omnivariance, anisotropy, eigentropy, normals, density and average height) and two radiometric features (average and maximum reflectivity) are extracted. The proposed method was applied to street point clouds to segment by Random Forest algorithm the classes road, curbs, sidewalks, building, vegetation, cars, poles and others. The average accuracy obtained with the structuring element was 81.2%, while based on KNN of 25 neighbors, the average accuracy was 82.2% and by means of a search radius, 88.2%. Consequently, it is deduced that although the segmentation is improved in some classes, the extraction of a high number of features (fourteen for each point of the structuring element) significantly hinders the training.

FRÉDÉRIC BARBARESCO

*Thales, France*

### **Symplectic Foliation Structures of Heat Theory and Information Geometry.**

We present a new symplectic model of Information Geometry [1,2] based on Jean-Marie Souriau's Lie Groups Thermodynamics [3,4]. Souriau model was initially described in chapter IV "Statistical Mechanics" of his book "Structure of dynamical systems" published in 1969. This model gives a purely geometric characterization of Entropy, which appears as an invariant Casimir function in coadjoint representation, characterized by Poisson cohomology. Souriau has proved that we can associate a symplectic manifold to coadjoint orbits of a Lie group by the KKS 2-form (Kirillov, Kostant, Souriau 2-form) in the affine case (affine model of coadjoint operator equivariance via Souriau's cocycle) [5], that we have identified with Koszul-Fisher metric from Information Geometry. Souriau established the generalized Gibbs density covariant under the action of the Lie group. The dual space of the Lie algebra foliates into coadjoint orbits that are also the Entropy level sets that could be interpreted in the framework of Thermodynamics by the fact that dynamics on these symplectic leaves are non-dissipative, whereas transversal dynamics, given by Poisson transverse structure, are dissipative. We will finally introduce Gaussian distribution on the space of Symmetric Positive Definite (SPD) matrices, through Souriau's covariant Gibbs density by considering this space as the pure imaginary axis of the homogeneous Siegel upper half space where  $\text{Sp}(2n, \mathbb{R})/\text{U}(n)$  acts transitively. We will also consider Gibbs density for Siegel Disk where  $\text{SU}(n, n)/\text{S}(\text{U}(n) \times \text{U}(n))$  acts transitively. Gauss density of SPD matrices is then computed through Souriau's moment map and coadjoint orbits.

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SILVIA BENVENUTI

*Alma Mater Studiorum Università di Bologna, Italy*

**TBA.**

NIREN BHOJA

*University of Nottingham, UK*

**Using octonions to understand spinors in 14 dimensions.**

Spinors in 14 dimensions can be understood using octonions. The goal of the talk is to introduce octonions in a way that makes them useful in calculations and the role they play in constructing representations in 8,10, 12, and 14 dimensions. There will be a discussion of how these relate to the classification of orbits of the group  $\text{Spin}(11,3)$  and how one can use these octonionic representations of spinors to understand the Standard Model of Physics.

ANDREAS CAP

*Universität Wien, Austria*

**On sub-Riemannian structures in low dimensions.**

I'll first discuss relations between sub-Riemannian structures,  $G$ -structures and filtered  $G$ -structures. I will then specialize to the three dimensional case, and discuss relations to parabolic geometries and BGG sequences in this case.

ALESSANDRO CAROTENUTO

*Masarykova univerzita, Czechia*

**Principal pairs of quantum homogeneous spaces.**

We introduce a simple but effective framework to build examples of principal comodule algebras, the quantum analogue of principal fibrations. Here the total space and the basis

of the fibration are quantum homogeneous spaces of a given Hopf algebra  $A$ , while the role of the structure group is played by a certain Hopf subalgebra of  $A$ . We also show how one can relax these assumptions in order to get a generalization of the usual notion of an Hopf–Galois extension that falls into the framework recently proposed by Brzezinski and Szymanski as Noncommutative analogues of fibrations with homogeneous fibre. In both cases, we will present a number of concrete examples with quantum flag manifolds.

ALBERTO CATTANEO

*Universität Zürich, Switzerland*

### **Poisson structures from corners of field theories.**

The BV formalism and its shifted versions in field theory have a nice compatibility with boundary structures. Namely, one such structure in the bulk induces a shifted (possibly degenerated) version on its boundary, which can be interpreted as a Poisson structure (up to homotopy). I will present the results for some field theories, in particular, 4D BF theory and 4D gravity.

TÜRKÜ ÖZLÜM ÇELİK

*Boğaziçi Üniversitesi, Turkey*

### **Algebraic Curves, Computer Algebra and Integrable Systems.**

Algebraic curves have found an important application in the context of integrable systems through the use of theta functions. These functions give rise to solutions of the Kadomtsev–Petviashvili hierarchy, a universal integrable system. In this talk, we will take a dive into this fascinating field from the perspective of computational algebraic geometry. Our focus will be on employing state-of-the-art tools in symbolic, numerical, and combinatorial algebraic geometry to explore these solutions and their applications back in the study of algebraic curves.

CATARINA P. COUTINHO

*Studio oculistico D'Azeglio, Bologna, Italy*

### **Machine learning application in visual field loss for Dominant Optic Atrophy.**

The purpose of this study was the identification of the characteristic visual field loss patterns of dominant optic atrophy (DOA) employing the Archetypal Analysis (AA) machine learning algorithm in Python 3.8. For 64 patients affected by molecularly confirmed DOA with OPA1 heterozygous mutation, binocular visual field (VF) tests performed by SITA standard 30-2 Humphrey VF analyser (Carl Zeiss Meditec, Dublin, CA, USA) were collected. When available, multiple VF tests from different time points were collected



to enlarge the dataset. Considering 229 VF test, preliminary results using AA detected archetypes (AT) for the characterisation of visual loss in DOA. Central ATs revealed to be the most significant ones, matching the common characteristic central or ceco-central scotoma for DOA patients, followed by the quadrantanopia, nasal step, and altitudinal ATs. The primary implementation of this algorithm focused on the DOA disease evidenced the potential to help in the identification and distinction of the typical visual loss patterns, which is in line with other works focused on diseases as glaucoma and idiopathic intracranial hypertension.

CARLO ALBERTO CREMONINI

*Università degli Studi dell'Insubria, Italy*

**TBA.**

ELISA ERCOLESSI

*Alma Mater Studiorum Università di Bologna, Italy*

**TBA.**

JORDAN FRANÇOIS

*Masarykova univerzita, Brno - Czechia*

**Twistors and conformal Cartan geometry as non-standard geometric structures.**

We present a conservative generalisation of vector bundles associated to a principal bundle  $P$ . These are constructed via 1-cocycles of the action of the structure group  $H$  on  $P$ . The adapted notion of connection on  $P$  needed for the covariant derivation of sections of these bundles naturally extends Ehresmann connections. These sections and connections are, in the physics parlance, "generalised gauge fields". As an illustration, we consider a way to obtain local twistors and their connection as end products of a gauge symmetry reduction of the conformal Cartan geometry: thus built, twistors and their connection appear as a clear instance of generalised gauge fields.

PATRIZIO FROSINI

*Alma Mater Studiorum Università di Bologna, Italy*

### **Recent advances in the theory of GENEOS and its application to Machine Learning.**

Group equivariant non-expansive operators (GENEOs) have been introduced a few years ago as mathematical tools for approximating data observers when data are represented by real-valued or vector-valued functions (<https://rdcu.be/bP6HV>). The use of these operators is based on the assumption that the interpretation of data depends on the geometric properties of the observers. In this talk we will illustrate some recent results in the theory of GENEOS, showing how these operators can make available a new approach to topological data analysis and geometric deep learning.

FABIO GAVARINI

*Università degli Studi di Roma Tor Vergata, Italy*

### **Real forms of complex Lie superalgebras and supergroups.**

A real form of a complex Lie algebra is the subset of fixed points of some "real structure", that is an antilinear involution; a similar description applies for real forms of complex (Lie or algebraic) groups. For complex Lie superalgebras, the notion of "real structure" extends in two different variants, called standard (a straightforward generalization) and graded (somewhat more sophisticated): the notion of "real form", however, stands problematic in the graded case. I will present the functorial version of "real structure" (standard or graded), and show that the notion of "real form" then properly extends, in both cases; along the same lines, I will introduce real structures and real forms for complex supergroups. Then, basing on a suitable notion of "Hermitian form" on complex superspaces, I will introduce unitary Lie superalgebras and supergroups (again standard or graded); any Lie superalgebra which embeds into a unitary one will then be called "super-compact" - and similar for supergroups. Finally, I will give nice existence/uniqueness results of super-compact real forms for complex Lie superalgebras which are simple of basic (or "contragredient") type, and similarly for their associated connected simply-connected supergroups. This is based on a joint work with Rita Fioresi, cf. *Comm. Math. Phys.* 397 (2023), no. 2, 937–965.

PAVLO GAVRYLENKO

*Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine and SISSA, Trieste, Italy*

**Isomonodromic tau functions on a torus, Fredholm determinants, and Nekrasov partition functions.**

I will consider the simplest example of isomonodromic deformations on a torus, which corresponds to the non-autonomous Calogero-Moser system. Solution and tau function of this system can be expressed either in terms of conformal blocks/Nekrasov partition functions, or in terms of some block Fredholm determinant. The latter one may also be generalized to the Fredholm determinant of a Riemann-Hilbert problem with arbitrary jump on a torus. The talk will be based on the joint paper with Fabrizio Del Monte and Harini Desiraju

<https://arxiv.org/pdf/2011.06292>.

ILJA GOGIĆ

*Sveučilišta u Zagrebu, Croatia*

**Applications of algebraic topology to operator algebras: Homogeneous  $C^*$ -algebras.**

The class of commutative  $C^*$ -algebras was fully described in the early days of the development of the theory of operator algebras: these are precisely algebras of continuous complex-valued functions on locally compact Hausdorff spaces that vanish at infinity. After commutative, the next simplest class are the homogeneous  $C^*$ -algebras, i.e.  $C^*$ -algebras whose all irreducible representations are of the same finite dimension. However, the problem of classification of such algebras becomes considerably more complicated due to certain topological phenomena which do not occur in the commutative case. In this talk we will present the overview of homogeneous  $C^*$ -algebras with an emphasis on some more recent results.

THEODORA IOANNIDOU

*Aristotle University of Thessaloniki, Greece*

**A brand new algebra.**

A novel discrete algebra is presented which follows by combining the  $SU(2)$  Lie-Poisson bracket with the discrete Frenet equation. Physically, the construction describes a discrete piecewise linear string in  $R^3$ . The starting point of our derivation is the discrete Frenet frame assigned at each vertex of the string. Then the link vector that connects the neighboring vertices is assigned the  $SU(2)$  Lie-Poisson bracket. Moreover, the same bracket defines the transfer matrices of the discrete Frenet equation which relates two

neighboring frames along the string. The procedure extends in a self-similar manner to an infinite hierarchy of Poisson structures. As an example, the first descendant of the  $SU(2)$  Lie-Poisson structure is presented in detail. For this, the spinor representation of the discrete Frenet equation is employed, as it converts the brackets into a computationally more manageable form. The final result is a nonlinear, nontrivial, and novel Poisson structure that engages four neighboring vertices.

ENNO KESSLER

*Max-Planck-Institut für Mathematik, Bonn, Germany*

### **An operadic structure on supermoduli spaces.**

The operadic structure on the moduli spaces of algebraic curves encodes in a combinatorial way how nodal curves in the boundary can be obtained by glueing smooth curves along marked points. In this talk, I will present a generalization of the operadic structure to moduli spaces of SUSY curves (or super Riemann surfaces). This requires colored graphs and generalized operads in the sense of Borisov- Manin.

Based joint work with Yu. I. Manin and Y. Wu. <https://arxiv.org/abs/2202.10321>

CAN KOZCAZ

*Boğaziçi Üniversitesi, Istanbul, Turkey*

### **On R-matrix formulation of $qq$ -characters.**

We introduce the R-matrix formulation of  $qq$ -characters and corresponding Frenkel-Reshetikhin deformed  $W$ -algebras. The R-matrix featuring in the construction is of the Ding-Iohara-Miki (DIM) algebra, while the type of the  $qq$ -character is determined by the network of Fock representations corresponding to a web of 5-branes geometrically engineering a quiver gauge theory. This formulation gives in a unified fashion the expressions for the characters of  $A_n$  and  $\hat{A}_n$  types. In the latter case we discover new  $qq$ -characters, which are Laurent series in the instanton parameter.

ANDREY KRUTOV

*Akademie věd České republiky, Prague, Czechia*

### **Transgressions in the relative Weil algebra.**

Let  $G$  be a Lie group and  $\mathfrak{g}$  be its Lie algebra.

The transgression map between invariants in the symmetric algebra  $S(\mathfrak{g}^*)$  of the dual space  $\mathfrak{g}^*$  and invariants in the exterior algebra  $\Lambda(\mathfrak{g}^*)$  of  $\mathfrak{g}^*$  was define by H. Cartan in 1950.

Later it appear in the work of Chern and Simons on the theory of  $G$ -principal bundles.

The transgression map can be constructed using cohomological properties of the Weil algebra  $W(\mathfrak{g})$  of  $\mathfrak{g}$ .

The Weil algebra  $W(\mathfrak{g}) = S(\mathfrak{g}^*) \otimes \Lambda(\mathfrak{g}^*)$  is a differential graded algebra introduced by A. Weil as an algebraic model for differential forms on the classifying bundle of  $G$ .

We generalise this construction to the relative case of symmetric spaces.

Let  $G$  be a real reductive Lie group and  $\mathfrak{g}$  be the complexification of its Lie algebra.

The Cartan involution on  $G$  induces the Cartan decomposition  $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$  of  $\mathfrak{g}$ . The corresponding relative Weil algebra is  $W(\mathfrak{g}, \mathfrak{k}) = (S(\mathfrak{g}) \otimes \Lambda(\mathfrak{p}))^{\mathfrak{k}}$ . We define the transgression map between  $\mathfrak{g}$ -invariants in  $S(\mathfrak{g})$  and  $\mathfrak{k}$ -invariants in  $\Lambda(\mathfrak{p})$  and study its properties.

This in joint work with Karmen Grizelj and Pavle Pandžić (Zagreb).

SYLVAIN LAVAU

*Aristotle University of Thessaloniki, Greece*

### **Singular foliations, a graded geometric approach.**

The notion of Singular Foliation most adapted for applicationACs to dynamical systems and geometry is the following: a locally finitely generated sheaf of vector fields closed under the Lie bracket. Their singular behaviour, however, makes it very challenging to advance the theory using classical differential geometry. Graded geometry on the other hand, offers a new class of objects that, although stated in more involved terms, are easier to manipulate and allows us to go beyond the difficulty raised by the singularities of the foliation. Thanks to these higher ‘more regular’ structures, it becomes possible to proceed to some computations that were otherwise very difficult to handle, or to generalize to the singular setting some notions that were until now only defined in the regular one. In this talk, we will explain how one can ‘replace’ a singular foliation by an adequate graded manifold, and how the Lie bracket on the former lifts to a Lie infinity-algebroid structure on the latter. We will then give several applications of this construction regarding deformations and characteristic classes of singular foliations.

MARIA A. LLEDÓ

*Universitat de València, Spain*

### **Communication and dissemination plan for CaLISTA.**

SARA LOMBARDO  
*Herriot Watt University, UK*

**TBA.**

ANDREA MALAGOLI  
*VST, Modena, Italy*

**TBA.**

ALESSIO MARRANI  
*Universidad de Murcia, Spain*

**The “magic star” projection: from exceptional structures to special Vinberg algebras.**

I will present the so-called “magic star” projection (also named “G2 decomposition” by Mukai), which allows to go "beyond" exceptional Lie algebras and cubic Jordan algebras, involving rank-3 special Vinberg T-algebras, and finding applications in supersymmetric Maxwell-Einstein gravity theories in various space-time dimensions.

BENJAMIN MCKAY  
*University College Cork, Ireland*

**Holomorphic Cartan geometries.**

I will review some complex algebraic geometry, the known holomorphic parabolic geometries on smooth projective varieties, our conjectures, and how much we can prove so far.

ALEXANDER MIKHAILOV  
*University of Leeds, UK*

**A novel approach to quantisation of dynamical systems.**

We propose to revisit the problem of quantisation and look at it from an entirely new angle, focussing on quantisation of dynamical systems themselves, rather than of their Poisson structures. We begin with a lift of a classical dynamical system to a system on a free

associative algebra with non-commutative dynamical variables and reduce the problem of quantisation to the problem of studying of two-sided quantisation ideals, i.e. the ideals of the free algebra that define the commutation relations of the dynamical variables and are invariant with respect to the non-commutative dynamics. Quantum multiplication rules in the quotient algebra over a quantisation ideal are manifestly associative and consistent with the dynamics. We found first examples of bi-quantum systems which are quantum counterparts of bi-Hamiltonian systems in the classical theory. Moreover, the new approach enables us to define and present first examples of non-deformation quantisations of dynamical systems. The new approach also sheds light on the problem of operator's ordering.

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IAN MUSSON

*University of Wisconsin-Milwaukee, US*

## On the geometry of algebras related to the Weyl groupoid.

Let  $\mathbf{k}$  be an algebraically closed field of characteristic zero. Let  $\mathfrak{g}$  be a finite dimensional classical simple Lie superalgebra over  $\mathbf{k}$  or  $\mathfrak{gl}(m, n)$ . In the case that  $\mathfrak{g}$  is a Kac-Moody algebra of finite type with set of roots  $\Delta$ , Sergeev and Veselov introduced the Weyl groupoid  $\mathfrak{W} = \mathfrak{W}(\Delta)$ , which has significant connections with the representation theory of  $\mathfrak{g}$ . Let  $\mathfrak{h}$ ,  $W$  and  $Z(\mathfrak{g})$  be a Cartan subalgebra of  $\mathfrak{g}_0$ , the Weyl group of  $\mathfrak{g}_0$  and the center of  $U(\mathfrak{g})$  respectively. Also let  $G$  be a Lie supergroup with  $\text{Lie } G = \mathfrak{g}$ . There are several important commutative algebras related to  $\mathfrak{W}$ . Namely

- The image  $I(\mathfrak{h})$  of the injective Harish-Chandra map  $Z(\mathfrak{g}) \rightarrow S(\mathfrak{h})^W$ .
- The supercharacter  $\mathbb{Z}$ -algebras  $J(\mathfrak{g})$  and  $J(G)$  of finite dimensional representations of  $\mathfrak{g}$  and  $G$ .

Let  $\mathcal{A} = \mathcal{A}(\mathfrak{g})$  be denote either  $I(\mathfrak{h})$  or  $J(G) \otimes_{\mathbb{Z}} \mathbf{k}$ . The purpose of this paper is to investigate the algebraic geometry of  $\mathcal{A}$ . In many cases, the algebra  $\mathcal{A}$  satisfies the Nullstellensatz. This gives a bijection between radical ideals in  $\mathcal{A}$  and superalgebraic sets (zero loci of such ideals). Any superalgebraic set is uniquely a finite union of irreducible superalgebraic components. In the non-exceptional Kac-Moody case, we describe the smallest superalgebraic set containing a given (Zariski) closed set, and show that the superalgebraic sets are exactly the closed sets that are unions of groupoid orbits.

KATHARINA NEUSSER

*Masarykova univerzita, Czechia*

### **Cone structures in differential and algebraic geometry.**

A cone structure on a complex manifold  $M$  is a closed submanifold  $\mathcal{C} \subset \mathbb{P}TM$  of the projectivized (holomorphic) tangent bundle of  $M$  which is submersive over  $M$ . Such structures arise naturally in differential and algebraic geometry, and when they do, they are typically equipped with a conic connection that specifies a distinguished family of curves on  $M$  in direction of  $\mathcal{C}$ . In differential geometry, a classical example is the null cone bundle of a holomorphic conformal structure with the conic connection given by the null-geodesics. In algebraic geometry, one has the cone structures consisting of varieties of minimal rational tangents (VMRT) induced by minimal rational curves on uniruled projective manifolds. In this talk we will discuss various examples of cone structures and will introduce two invariants for conic connections. As an application of the study of these invariants, we will present a local-differential-geometric version of the global algebraic-geometric rigidity theorem of Mok and Hong–Hwang, which recognizes certain generalized flag varieties from their VMRT-structures. This talk is based on joint work with Jun-Muk Hwang.

SIMONE NOJA

*Universität Heidelberg, Germany*

### **Nilpotence Varieties, Pure Spinors Superfields and Supersymmetry.**

In this talk I will introduce a (super-)mathematical perspective on the pure spinor superfield formalism, showing how to recover supersymmetry multiplets from geometric data related to the nilpotence variety of a certain Poincaré superalgebra. After discussing some lower dimensional examples, I will focus on the case of supersymmetry in six dimensions, where the nilpotence variety is a Segre manifold, and I will hint at a generalization of the formalism in the direction of derived algebraic geometry. If time permits, I will discuss how nilpotence varieties of some classical Lie superalgebras are related to the superconformal field theories.

RÉAMONN Ó BUACHALLA

*Univerzita Karlova, Czechia*

### **Quantum Flag Manifolds and Noncommutative Geometric Representation Theory.**

We present recent progress on noncommutative geometric representations of quantum algebraic objects, such as Drinfeld-Jimbo modules, Nichols algebras, and quantum BGG



sequences. The noncommutative geometry underlying these realisations is a  $q$ -deformed Dolbault complex for the A-series quantum flag manifolds. This complex is built in a very natural way from Lusztig's quantum root vectors, and is shown to be quite sensitive to the required choice of reduced decomposition of the longest element of the Weyl group. When these constructions are restricted to the quantum Grassmannians, they coincide with prior research on the celebrated Heckenberger-Kolb calculus.

CHIARA PAGANI

*Alma Mater Studiorum Università di Bologna, Italy*

**TBA.**

MAURIZIO PARTON

*Università degli studi G. D'Annunzio Chieti Pescara, Pescara, Italy*

### **Introduction to geometric deep learning.**

The last few years have witnessed the impressive success of deep learning in several very different fields: image recognition, games, biology, and natural language processing, to name a few. However, we are still far from truly understanding the mathematics behind these accomplishments. Besides being of a theoretical interest by itself, understanding why these techniques work so well would certainly increase both their performance and fields of application. Geometric deep learning (GDL) is a promising field targeting such a mathematical understanding. GDL proposes a unified approach to understanding why diverse architectures, such as CNN, LSTM, Graph Neural Networks, and Transformers, are so successful. The underlying powerful idea is that whenever the function to be approximated is invariant by a group  $G$  of symmetries,  $G$ -invariance should be encoded in the architecture. For instance, this is why CNNs, which implement a translational invariance, work so well on image recognition, which is a task naturally translation-invariant. In this talk I will sketch an introduction to this marvellous topic.

ANDREA SANTI

*Università degli studi di Roma Tor Vergata, Italy*

### **On 3-nondegenerate CR manifolds in dimension 7.**

I will report on joint works with B. Kruglikov on CR hypersurfaces in  $C^4$  with a degenerate Levi form. I will discuss the symmetry dimension bound 8 for all the 3-nondegenerate 7-dimensional CR real-analytic structures and present the classification of the locally homogeneous ones. The bound 8 is achieved on the homogeneous model, which is locally the only homogeneous 3-nondegenerate 7-dimensional CR manifold.

KAREN STRUNG

*IMCAS, Czechia*

**TBA.**

ALESSANDRO TANZINI

*SISSA, Trieste, Italy*

**tt\* Toda equations and supersymmetric gauge theories.**

We show how the study of supersymmetric gauge theories in presence of surface defects allows to formulate and solve Hirota bilinear relations for tt\* Toda equations for general simple group.

DAVID TAYLOR

*Logica Capital, US*

**Hamilton Monte Carlo on symmetric spaces via Cartan geometry.**

The celebrated Hamilton Monte Carlo (HMC) algorithm is a powerful tool in computational statistics for sampling from intractable probability distributions. While HMC on Euclidean space is well-developed, implementing HMC on more general manifolds remains a major challenge. We propose an efficient HMC algorithm for symmetric spaces, employing Cartan geometry and reduction theory for Hamiltonian dynamics on a principal bundle. We validate our findings by implementation on a simulated dataset.

JURAI TEKEL

*Univerzita Komenského v Bratislave, Slovakia*

**Why to talk and how to talk to general public about science.**

Science is important for society. Even though there are very few people who would argue with this statement, it is important to remind people about this and the role of knowledge in the quality of life we enjoy today. This is one of the roles of public outreach. The others are to promote the career of a scientist among young people, to attract bright people to STEM jobs and to encourage all people to be curious and eager to discover things about the world around them. Because one can enjoy the knowledge about nature without being a scientist the same way one can enjoy riding their bicycle without aspiring to win the Tour. In this talk, I will give a few examples of public outreach activities we have been

involved in Slovakia and abroad and I will also mention some of my favorite topics of talks and activities for the general public.

SVJETLANA TERZIĆ

*Univerzitet Crne Gore, Montenegro*

### **The spaces of parameters for $T^n$ -action on $G_{n,2}$ and the moduli spaces $\overline{\mathcal{M}}_{0,(t_1,\dots,t_n)}$ .**

The canonical action of the compact torus  $T^n$  on a complex Grassmann manifold  $G_{n,2}$  of two-dimensional complex subspaces in  $\mathbb{C}^n$  is an important and widely known example which appears to many areas of mathematics.

The main stratum  $W_n \subset G_{n,2}$  given by those points from  $G_{n,2}$  whose all Plücker coordinates are non-zero, plays a crucial role in description of the space  $G_{n,2}/T^n$ , as it is an open dense set in  $G_{n,2}$  and it belongs to any Plücker chart for  $G_{n,2}$ . In addition, we earlier proved that  $W_n \cong \overset{\circ}{\Delta}_{n,2} \times F_n$ , where  $\Delta_{n,2}$  is the hypersimplex and  $F_n = W_n/(\mathbb{C}^*)^n \subset \mathbb{C}P^N$ ,  $N = \binom{n-2}{2}$  is an open algebraic manifold. In order to construct a model for the orbit space  $G_{n,2}/T^n$  it turns out to be important to look for a compactification  $\mathcal{F}_n$  for  $F_n$  such that any automorphism of  $F_n$  induced by the transition maps between the Plücker charts extends to the automorphism of  $\mathcal{F}_n$ . Such compactification  $\mathcal{F}_n$  we call the universal space of parameters. We obtain the space  $\mathcal{F}_n$  by resolving singularities that arise, using the techniques of wonderful compactification from algebraic geometry.

On the other side, the moduli space  $\overline{\mathcal{M}}_{0,n}$  of  $n$ -pointed stable genus zero curves is the Deligne-Mumford-Grothendieck-Knudsen compactification of the moduli space  $\mathcal{M}_{0,n}$  of  $n$ -pointed genus zero curves. The space  $\overline{\mathcal{M}}_{0,n}$  is proved by Kapranov to coincide with the Chow quotient  $G_{n,2}/(\mathbb{C}^*)^n$ . By proving that  $\mathcal{F}_n$  coincides with  $\overline{\mathcal{M}}_{0,n}$  we present another approach to the compactification of  $\mathcal{M}_{0,n}$ .

In addition, we discuss the relations between the spaces of parameters  $F_\omega$  of the chambers  $C_\omega \subset \Delta_{n,2}$ , which arise from the matroidal decomposition of  $\Delta_{n,2}$ , and the moduli space  $\overline{\mathcal{M}}_{0,(t_1,\dots,t_n)}$  of weighted  $n$ -pointed stable genus zero curves, where  $(t_1, \dots, t_n) \in C_\omega$ .

The talk is based on the results obtained jointly with Victor M. Buchstaber.

DENNIS THE

*UiT Norges arktiske universitet, Tromsø, Norway*

### **Exceptionally simple super-PDE.**

Cartan and Engel gave the first geometric realisations of the (complex) exceptional simple Lie algebra  $G_2$ , namely as the symmetries of various differential geometric structures. I'll describe generalizations of this story to the other exceptional simple Lie algebras, and more recently the exceptional simple Lie superalgebras  $G(3)$  (joint work with Kruglikov and Santi) and  $F(4)$  (joint work with Santi).

MILOSLAV TORDA

*University of Liverpool, UK*

## **Maximally Dense Crystallographic Symmetry Group Packings through Entropic Trust Region: An Information Geometric Perspective..**

Stochastic relaxation is a well-established technique in machine learning and artificial intelligence used to tackle complex optimization landscapes. Here, we employ stochastic relaxation to address the challenge of discovering the densest packing of closed subsets of  $n$ -dimensional Euclidean space, subject to constraints imposed by the Crystallographic Symmetry Group (CSG). To this end, we introduce the Entropic Trust Region Packing Algorithm (ETRPA), which is an instance of the natural gradient learning method with adaptive selection quantile fitness rewriting. Since CSGs induce a toroidal topology on the configuration space, we perform the ETRPA search on a parametric family of probability measures defined on an  $n$ -dimensional torus. We examine the information geometry of the ETRPA through its equivalence with the generalized proximal method and characterize the algorithm via local dual geodesic flows that maximize multi-information, a measure of stochastic dependence in complex systems. Therefore, the evolutionary dynamics, statistical physics, and information theoretic background of the ETRPA can be understood in terms of more general graphical interaction models. This work is motivated by the problem of molecular Crystal Structure Prediction, which involves predicting a synthesizable periodic structure based on a molecule's chemical composition and specific pressure-temperature conditions.

KOSTADIN TRENCEVSKI

*Ss. Cyril and Methodius University, Skopje, Macedonia*

## **Global scheme of the basic interactions and their geometrical interpretations.**

Our space-time consists of three 3-dimensional spaces: space  $S$ , space rotations  $SR$  and time  $T$ . First are considered the basic possible 4 cases for exchange among them: 1.  $r \rightarrow s$ , 2.  $s \rightarrow r$ , 3.  $r \rightarrow t$ , and 4.  $s \rightarrow t$ , where  $s \in S$ ,  $r \in SR$ , and  $t \in T$ . Analogous to the affine group of translations and rotations  $\mathcal{A}$ , it is considered a space group  $G_s$  of  $6 \times 6$  matrices, which is isomorphic to the group  $Spin(4)$ . The space metric observed by the particles is found. Further are considered 4 generalized exchanges  $1^*$ ,  $2^*$ ,  $3^*$  and  $4^*$ , induced by the cases 1, 2, 3, and 4. The case  $1^*$  leads to the electro-weak interaction, and it is a consequence of non-commutativity between one translation and one rotation in the space group  $G_s$ . The case  $2^*$  leads to the strong interaction, and it is a consequence of non-commutativity between two translations in the space group  $G_s$ . It leads also to the galactic acceleration which is observed at the periphery of each galaxy, and now we do not need dark matter in order to explain the motion of the distant stars in the galaxies. The case  $3^*$  leads to electromagnetic interaction, and it is a consequence of non-commutativity between one translation and one rotation in the affine group  $\mathcal{A}$ . The case  $4^*$  leads to gravitational interaction and it is a consequence of non-commutativity between

one translation and one "radial translation" in the affine group  $\mathcal{A}$ . The corresponding accelerations are deduced and for a fixed space positions they are of type  $\mathbf{a} = \text{rot}(\vec{\varphi})$  (gauge invariant), but the quantum and wave effects are neglected. It is also predicted a new gravity-weak interaction, which belongs to the case 2\*.

ELIOT TRON

*Ecole Nationale de l'Aviation Civile, France*

### **Canonical foliations of neural networks: application to robustness.**

Adversarial attacks are emerging as threats to the trustability of machine learning. Understanding these attacks is becoming a crucial task. We propose a new vision on neural network robustness using Riemannian geometry and foliation theory. The idea is illustrated by creating a new adversarial attack that takes into account the curvature of the data space. This new adversarial attack called the two-step spectral attack is a piecewise linear approximation of a geodesic in the data space. The data space is treated as a (pseudo) Riemannian manifold equipped with the pullback of the Fisher Information Metric (FIM) of the neural network. In most cases, this metric is only semi-definite and its kernel becomes a central object to study. A canonical foliation is derived from this kernel. The curvature of transverse leaves gives the appropriate correction to get a two-step approximation of the geodesic and hence a new efficient adversarial attack.

JING PING WANG

*University of Kent, UK*

### **Quantisations of Integrable Systems: the Volterra and Toda hierarchies.**

In this talk, we'll discuss in detail quantisations for two nonabelian hierarchies, namely the Volterra and the Toda hierarchies, applying a novel approach based on the notion of quantisation ideals. We prove that the nonabelian Volterra together with the whole hierarchy of its symmetries admit a deformation quantisation, and that all odd-degree symmetries of the Volterra hierarchy admit also a non-deformation quantisation. For the Toda hierarchy we find a bi-quantum structure, which can be regarded as a quantum deformation of its classical bi-Hamiltonian structure. The problem of finding the Hamiltonians of the quantised systems will also be discussed. This is a joint work with S. Carpentier and A.V. Mikhailov.

THOMAS WEBER

*Università di Torino, Italy*

### **Infinitesimal braidings and pre-Cartier categories.**

Infinitesimal braidings are natural transformations in symmetric categories which can be used to construct braidings on the formal power series expansion of the category. Most prominently, for semisimple Lie algebras  $\mathfrak{g}$  this leads to a braiding on formal power series of  $U\mathfrak{g}$ -modules which is equivalent to that of the corresponding Drinfeld-Jimbo quantum group. In this talk we propose a more general approach to infinitesimal braidings which applies to arbitrary braided categories. The motivating idea is to understand such an infinitesimal braiding as a first order deformation of a given braiding. We call such categories pre-Cartier, as they generalize previously studied Cartier categories. In case of quasitriangular bialgebra (co)modules we discuss the algebraic structure equivalent to an infinitesimal braiding. These are Hochschild 2-cocycles iff a deformed version of the quantum Yang-Baxter equation holds. We discuss several examples of infinitesimal braidings, particularly on  $q$ -deformed  $SL(N)$ , Sweedler's Hopf algebra and via twisting. As main results we provide an infinitesimal FRT construction and Tannaka-Krein reconstruction theorem for pre-Cartier bialgebras. The former admits canonical non-trivial solutions and consequently induces infinitesimal braidings on several classes of quantum homogeneous spaces. The talk is based on a collaboration with Ardizzoni, Bottegoni and Sciandra.

HENRIK WINTHER

*Masarykova univerzita, Czechia*

### **Jets and differential operators in noncommutative geometry.**

We construct an infinite family of endofunctors on the category of left modules over a unital associative algebra equipped with a differential calculus. These functors generalize the jet functors on vector bundles from differential geometry. In particular, our construction coincides with the classical jet functor for vector bundles when the algebra is the smooth functions on a manifold and the calculus is generated by the classical exterior derivative. We show that our jet functors gives rise to a category of linear differential operators between modules, that these satisfy many good properties one might expect, and that most maps which are expected to be differential operators (connections, differentials, partial derivatives) are. We also discuss representability, symbols and define a notion of vector fields in this setting. Joint work with K. Flood and M. Mantegazza.

FERDINANDO ZANCHETTA

*Alma Mater Studiorum Università di Bologna, Italy*

### **Geometric Deep Learning: from theory to applications.**

Geometric Deep Learning (GDL) is a recent emerging part of Machine Learning blending together ideas coming from Geometry and Deep Learning. After an introduction to the mathematical foundations of the subject, I will speak about some applications of GDL, explaining some current research directions.

FABRIZIO ZANELLO

*Georg-August-Universität Göttingen, Germany*

### **Renormalization of Higher Currents of the sine-Gordon Model in pAQFT.**

The goal of the talk is to show that the higher currents of the sine-Gordon model are renormalizable by finite renormalization in the framework of pAQFT. First we consider the classical sine-Gordon model and use techniques from the theory of integrable systems to obtain closed recursive formulas for the conserved currents. We then move to the pAQFT setting and perform what we call the piecewise renormalization of the interacting currents, namely the renormalization of the elementary parts of the unrenormalized expressions separately. Finally we prove that reassembling the piecewise renormalized elementary parts all together produces a well-defined result.