

Weyl groupoids, Young tableaux and Borel subalgebras.

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Abstract

Let \mathbf{k} be an algebraically closed field of characteristic zero. Let \mathfrak{g} be the Lie superalgebra $\mathfrak{sl}(n|m)$. Let \mathfrak{W} be the Weyl groupoid introduced by Sergeev and Veselov [SV11] using the root system of \mathfrak{g} . In [SV17], motivated by deformed quantum Calogero-Moser problems [SV04], the same authors considered action of \mathfrak{W} on $\mathbf{k}^{n|m}$ depending on a parameter κ . When κ is negative special, they showed this action has infinite orbits. In the case $m > n$, with m, n relatively prime we study a particular infinite orbit with some special properties. This orbit, thought of as an directed graph is isomorphic to the graph of an orbit of the root groupoid for the affinization $L(\mathfrak{g})$ of \mathfrak{g} . This groupoid has a base consisting of Borel subalgebras, and morphisms are given by odd reflections.

The underlying reason for this graph isomorphism is that both have the same combinatorics. This combinatorics involves Young diagrams and tableaux drawn on the surface of a rotating cylinder with circumference m and length n . We introduce the *Young groupoid* which has the set of such diagrams, and has morphisms that correspond to adding or deleting a box from the diagrams. Allowing the cylinder to repeat produces an infinite orbit. This leads to a third graph which is isomorphic to the other two.

References

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