

NONPUB
NONLOCAL AND NONLINEAR PDES
AT THE UNIVERSITY OF BOLOGNA

September 8-9, 2022

WORKSHOP BOOKLET

Venue

All talks will take place at the Mathematics Department of the Alma Mater Studiorum Università di Bologna, in Piazza di Porta S. Donato 5, Bologna.

The room is **Aula Vitali**, on the ground floor.

Abstracts

Decay estimates in evolution equations with classical and fractional time-derivatives

Elisa Affili

Alma Mater Studiorum Università di Bologna

☉ Sep 8, 11:00

Using energy methods, we prove some power-law and exponential decay estimates for classical and nonlocal evolutionary equations. The results obtained are framed into a general setting, which comprise, among the others, equations involving both standard and Caputo time-derivative, and diffusion operators as the classic and fractional Laplacian, complex valued magnetic operators, fractional porous media equations and nonlocal Kirchhoff operators. Both local and fractional space diffusion are taken into account, possibly in a nonlinear setting. The different quantitative behaviours, which distinguish polynomial decays from exponential ones, depend heavily on the structure of the time-derivative involved in the equation. This work was done in collaboration with Enrico Valdinoci [1].

- [1] E. Affili and E. Valdinoci. Decay estimates for evolution equations with classical and fractional time-derivatives. *Journal of Differential Equations*, 266(7):4027–4060, 2019.

A Brezis-Nirenberg type result for mixed local and nonlocal operators

Stefano Biagi

Politecnico di Milano

☉ Sep 8, 12:00

In this seminar we present some existence results, in the spirit of the celebrated paper by Brezis and Nirenberg (CPAM, 1983), for a *critical problem* driven by a mixed local and nonlocal linear operator. More precisely, given a bounded open set $\Omega \subseteq \mathbb{R}^n$ (with $n \geq 4$) and $\lambda > 0$, we consider the *perturbed critical problem*

$$(P) \quad \begin{cases} -\Delta u + (-\Delta)^s u = u^{\frac{n+2}{n-2}} + \lambda u^p & \text{in } \Omega \subseteq \mathbb{R}^n, \\ u \geq 0 & \text{in } \Omega, \\ u = 0 & \text{in } \partial\Omega \end{cases}$$

and we develop an existence theory, both in the case of linear (that is, $p = 1$) and superlinear (that is, $p > 1$) perturbations. In the particular case $p = 1$, we also investigate the mixed Sobolev inequality associated with (P), detecting the optimal constant, which we show that is never achieved.

The results discussed in this talk are obtained in collaboration with S. Dipierro, E. Valdinoci, and E. Vecchi.

Functions of least $W^{s,1}$ -fractional seminorm

Claudia Bucur

Università degli Studi di Milano

☉ Sep 9, 10:00

We discuss some properties of minimizers of the $W^{s,1}$ -fractional seminorm, that reflect those enjoyed by *functions of least gradient*, their classical counterparts. In particular, we investigate the connection between these minimizers and nonlocal minimal sets and take advantage of this connection to show the existence of *functions of least $W^{s,1}$ -seminorm*. We further reason about existence of minimizers and weak solutions by considering the asymptotics as $p \rightarrow 1$ of the $W^{s,p}$ -fractional seminorm and of its Euler-Lagrange equation. The results presented are obtained in collaboration with S. Dipierro, L. Lombardini, J. Mazón, and E. Valdinoci.

Blowing-up solutions for a nonlocal Liouville type equation in a union of intervals

Matteo Cozzi

Università degli Studi di Milano

☉ Sep 9, 14:30

We consider the nonlocal Liouville type equation $(-\Delta)^{\frac{1}{2}}u = \varepsilon\kappa e^u$ in the union I of $d \geq 2$ disjoint bounded intervals, coupled with homogeneous Dirichlet exterior data. Here, κ is a smooth bounded function with positive infimum and $\varepsilon > 0$ is a small parameter. For any integer $m \in [1, d]$, we construct a family of solutions $\{u_\varepsilon\}$ which blows up at m distinct interior points of I and for which $\varepsilon \int_I \kappa e^{u_\varepsilon} dx \rightarrow 2m\pi$ as $\varepsilon \searrow 0$. Moreover, we show that, when $d = 2$ and m is suitably large, no such construction is possible.

The talk is based on a joint work with Antonio J. Fernández (ICMAT, Madrid).

Some fourth order problems arising in Physics

Martina Magliocca

Université d'Évry-Val d'Essonne

☉ Sep 9, 12:00

We will see some fourth order problems arising in Physics which model different processes, such as the growth of crystal surfaces and wetting-dewetting processes. Mathematically speaking, we will focus on global existence and regularity results for problems as

$$\begin{cases} u_t = F(t, x, u, \nabla u, \dots, \Delta^2 u) & \text{in } [0, T] \times \mathbb{T}^N, \\ u(0, x) = u_0(x) & \text{in } \mathbb{T}^N, \end{cases}$$

where $\mathbb{T}^N = [-\pi, \pi]^N$ is the N -dimensional torus and the initial data u_0 belong to Wiener spaces. The particular choices of F will describe the model in object.

These results are contained in a joint work with R. Granero Belinchón [1], and in [2].

- [1] Granero-Belinchón, R. & Magliocca, M. (2019). Global Existence and Decay to equilibrium for some crystal surface models. *Discrete & Continuous Dynamical Systems-A*, 39(4), 2101-2131.
- [2] Magliocca, M. (2022). On a fourth order equation describing single-component film models. arXiv preprint arXiv:2203.13707.

Nonlocal Neumann boundary conditions

Edoardo Proietti Lippi

Università degli Studi di Firenze

☉ Sep 9, 11:00

We present some properties of a nonlocal version of the Neumann boundary conditions associated to problems involving the fractional p -Laplacian. For this problems, we show some regularity results for the general case and some existence results for particular types of problems. When $p = 2$, we give a generalization of the boundary conditions in which both the nonlocal and the classic Neumann conditions are present, and we consider problems involving both nonlocal and local interactions.

Rigidity results for the critical p -Laplace equation

Alberto Roncoroni

Politecnico di Milano

☉ Sep 8, 15:30

The starting point of the seminar is the well-known generalized Lane-Emden equation

$$\Delta_p u + |u|^{q-1}u = 0 \quad \text{in } \mathbb{R}^n, \quad (1)$$

where Δ_p is the usual p -Laplace operator with $1 < p < n$ and $q > 1$. I will discuss several non-existence and classification results for positive solutions of (1) in the subcritical ($q < p^* - 1$) and in the critical case ($q = p^* - 1$). In the critical case, it has been recently shown, exploiting the moving planes method, that positive solutions to the critical p -Laplace equation (i.e. (1) with $q = p^* - 1$) and with finite energy, i.e. such that $u \in L^{p^*}(\mathbb{R}^n)$ and $\nabla u \in L^p(\mathbb{R}^n)$, can be completely classified. In this talk, I will present some recent classification results for positive solutions to the critical p -Laplace equation with (possibly) infinite energy satisfying suitable conditions at infinity. Moreover, if time permits I will discuss analogue results in the anisotropic, conical and Riemannian settings.

This is based on a recent joint work with G. Catino and D. Monticelli.

Maximum principles and related problems for a class of nonlocal extremal operators

Delia Schiera

Instituto Superior Técnico, Universidade de Lisboa

☹ Sep 8, 14:30

I will consider a class of degenerate nonlinear operators that are extremal among operators with one dimensional fractional diffusion and that approximate the so-called truncated Laplacians. I will show some properties of these operators, emphasizing the differences both with the local equivalent operators and with more standard nonlocal operators such as the fractional Laplacian. In particular, continuity properties, validity of comparison and maximum principles, and their relation with principal eigenvalues, will be presented.

Joint work with Isabeau Birindelli and Giulio Galise.

Programme

	Thursday 8 Sep	Friday 9 Sep
10:00 – 10:55		Bucur
11:00 – 11:55	Affili	Proietti Lippi
12:00 – 12:55	Biagi	Magliocca
13:00 – 14:30	<i>Lunch break</i>	
14:30 – 15:25	Schiera	Cozzi
15:30 – 16:25	Roncoroni	
19:30	<i>Social dinner</i>	

Social dinner

The social dinner will be held on Thursday September 8 at

Trattoria Osteria Buca Manzoni

located in via Manzoni 6g, Bologna. It is at walking distance from the Mathematics Department, in the city centre.

We meet at **19:30** directly in front of the restaurant.