Rigidity results for the critical $p-$Laplace equation

Alberto Roncoroni

Politecnico di Milano

alberto.roncoroni@polimi.it

The starting point of the seminar is the well-known generalized Lane-Emden equation

$$\Delta_p u + |u|^{q-1}u = 0 \quad \text{in } \mathbb{R}^n\,, \tag{1}$$

where Δ_p is the usual $p-$Laplace operator with $1 < p < n$ and $q > 1$. I will discuss several non-existence and classification results for positive solutions of (1) in the subcritical ($q < p^* - 1$) and in the critical case ($q = p^* - 1$). In the critical case, it has been recently shown, exploiting the moving planes method, that positive solutions to the critical $p-$Laplace equation (i.e. (1) with $q = p^* - 1$) and with finite energy, i.e. such that $u \in L^{p^*}(\mathbb{R}^n)$ and $\nabla u \in L^p(\mathbb{R}^n)$, can be completely classified. In this talk, I will present some recent classification results for positive solutions to the critical $p-$Laplace equation with (possibly) infinite energy satisfying suitable conditions at infinity. Moreover, if time permits I will discuss analogue results in the anisotropic, conical and Riemannian settings.

This is based on a recent joint work with G. Catino and D. Monticelli.