

NONPUB24
NONLOCAL AND NONLINEAR PDES
AT THE UNIVERSITY OF BOLOGNA

June 6-7, 2024

WORKSHOP BOOKLET

Abstracts

An area formula for intrinsic regular graphs in homogeneous groups

Francesca Corni

Alma Mater Studiorum Università di Bologna

☉ June 6, 11:30

In this talk we present an explicit area formula to compute the spherical Hausdorff measure of an intrinsic regular graph in an arbitrary homogeneous group. In particular, we assume the intrinsic graph to be intrinsically differentiable at any point with continuous intrinsic differential. The main novelty of the area formula lies in the introduction of a suitable notion of intrinsic Jacobian and in the computation of an explicit expression for this object. This is joint work with V. Magnani (Università di Pisa).

A Γ -convergence result for 2D type-I superconductors

Alessandro Cosenza

Université Paris Cité

☉ June 6, 12:15

In the Ginzburg-Landau model superconductors are characterized by a parameter κ called the Ginzburg-Landau parameter. If $\kappa < \frac{1}{\sqrt{2}}$ the superconductors are classified as type-I, if $\kappa > \frac{1}{\sqrt{2}}$ they are classified as type-II. While in type-II superconductors vortices appear, in type-I superconductors normal and superconducting regions are formed, separated by interfaces. In particular by the Meissner Effect, if ρ is the density of superconducting electrons and B is the magnetic field, it is observed that $\rho B \simeq 0$. Considering a 3D sample, it is experimentally observed that complex patterns appear at the surface. It is believed that

these patterns are a manifestation of branching patterns inside the sample. In [2] two regimes of parameters for the 3D type-I model are derived, corresponding to uniform and non uniform branching patterns. Moreover, in [1] a Γ -convergence result is proved for the full 3D model in the case of uniform branching patterns. In this talk I present a Γ -convergence result for the 2D type-I Ginzburg-Landau model in the crossover of the two regimes found in [2]. This is a first step in understanding how to extend the results of [1] to the second regime. With these hypothesis on the parameters the energy functional shares similarities with a Modica-Mortola type functional and in the limit Γ -converges to the area functional. To prove this result, it is necessary to carefully treat the global interaction between the phase of the complex order parameter u and the vector potential A , taking into account the gauge invariance satisfied by the functional. This talk is based on an ongoing work with Michael Goldman and Alessandro Zilio.

- [1] S. Conti, M. Goldman, F. Otto, and S. Serfaty, *A branched transport limit of the Ginzburg-Landau functional*, J. Éc. polytech. Math. **5** (2018), 317–375.
- [2] S. Conti, F. Otto, and S. Serfaty, *Branched microstructures in the Ginzburg-Landau model of type-I superconductors*, SIAM J. Math. Anal. **48** (2016), no. 4, 2994–3034.

Spreading phenomena under singular potentials: statics and dynamics

Riccardo Durastanti

Università degli Studi di Napoli Federico II

☹ June 7, 11:30

We look at spreading phenomena under the action of singular potentials modelling repulsion between the liquid-gas interface and the substrate. We mainly discuss the static case: depending on the form of the potential, the macroscopic profile of equilibrium configurations can be either droplet-like or pancake-like,

with a transition profile between the two at zero spreading coefficient. These results generalize, complete, and give mathematical rigor to de Gennes' formal discussion of spreading equilibria. Uniqueness and non-uniqueness phenomena are also discussed. Then we will briefly focus on the dynamics, assuming zero slip-page at the contact line. Based on formal analysis arguments, we report that generic travelling-wave solutions exist and have finite rate of dissipation, indicating that singular potentials stand as an alternative solution to the contact-line paradox. In agreement with equilibrium configurations, travelling-wave solutions have microscopic contact angle equal to $\pi/2$ and, for mild singularities, finite energy.

This is a joint work with Lorenzo Giacomelli.

Rigidity and symmetry results for some elliptic problems

Francesco Esposito

Università della Calabria

☉ June 6, 17:15

In this talk, we investigate qualitative properties of singular solutions to some elliptic problems. In the first part, we will focus our attention on semilinear and quasilinear elliptic problems under zero Dirichlet boundary conditions. In the second part, thanks to the previous analysis, we obtain some rigidity results for overdetermined boundary value problems for singular solutions in bounded domains.

Mean value formulas for surfaces in Grushin spaces

Valentina Franceschi

Università degli Studi di Padova

☉ June 7, 12:15

In this talk, we consider n -dimensional Grushin spaces, where a Riemannian metric degenerates along a line in the space, resulting in a sub-Riemannian structure. We discuss the validity

of (sub-)mean value property for (sub-)harmonic functions on hypersurfaces within Grushin spaces of dimension $n > 2$. Our interest is driven by the classical counterpart: mean value formulas for harmonic functions on surfaces in the Euclidean setting are crucial for establishing the Bombieri-De Giorgi-Miranda gradient bound, which, in turn, plays a central role in the classical regularity theory. We conclude by presenting remarks and open questions about the regularity theory of minimal surfaces within this sub-Riemannian framework, which is yet to be established.

Aspects of total variation and connections with image processing

Serena Guarino Lo Bianco

Università degli Studi di Modena e Reggio Emilia

☺ June 6, 16:30

In [1] a BMO-type seminorm involving ε -cubes of general orientation was introduced in order to characterize sets of finite perimeter. Later, in [2] a more general representation formula for SBV functions was given. For BV functions a such representation does not exist. Inspired by [3] we study the connections between a BMO-type non-local functional and the total variation. Moreover we study a minimization problem based on the total variation image denoising model of Rudin, Osher, and Fatemi. Based on joint works with R. Schiattarella.

- [1] L. Ambrosio, J. Bourgain, H. Brezis, and A. Figalli, *BMO-type norms related to the perimeter of sets*, Comm. Pure Appl. Math. **69** (2016), no. 6, 1062–1086.
- [2] N. Fusco, G. Moscarillo, and C. Sbordone, *A formula for the total variation of SBV functions*, J. Funct. Anal. **270** (2016), no. 1, 419–446.
- [3] H. Brezis and H.-M. Nguyen, *Non-local functionals related to the total variation and connections with image processing*, Ann. PDE **4** (2018), no. 1, Paper No. 9, 77.

The Grushin eigenvalue problem: sensitivity, optimization, and blow-up

Paolo Luzzini

Università degli Studi del Piemonte Orientale Amedeo Avogadro

☉ June 6, 15:15

One of the oldest and most studied problems in the spectral theory of differential operators is the eigenvalue problem for the Dirichlet Laplacian. Classical questions about Laplacian eigenvalues concern their sensitivity analysis, optimization, asymptotic expansions, and many other more properties.

On the other hand, similar questions remain open for an important class of degenerate operators, that is the Grushin Laplacians. In this talk I will present some recent results regarding the spectral theory of the Grushin Laplacian and in particular its shape sensitivity analysis, the optimization of the first eigenvalue, and a blow-up analysis.

The sharp quantitative isocapacitary inequality

Michele Marini

Università degli Studi del Sannio

☉ June 6, 14:30

The well-known isocapacitary inequality states that balls minimize the capacity among all sets of the same given volume. In the talk, we prove a sharp quantitative form of this classical result. Namely, we show that the difference between the capacity of a set and that of a ball with the same volume bounds the square of the Fraenkel asymmetry of the set. We then discuss some possible extensions.

**Regularity for almost minimizer of a one-phase
Bernoulli-type functional in Carnot Groups of step
two**

Enzo Maria Merlino

Alma Mater Studiorum Università di Bologna

☉ June 7, 09:30

The regularity of minimizers of the classical one-phase Bernoulli functional was deeply studied after the pioneering work of Alt and Caffarelli. More recently, the regularity of almost minimizers was investigated as well. We present a regularity result for almost minimizers for a one-phase Bernoulli-type functional in Carnot Groups of step two. Our approach is inspired by the methods introduced by De Silva and Savin in the Euclidean setting. Moreover, some recent intrinsic gradient estimates have been employed. Generalizations to the nonlinear framework will be discussed. Some of the results presented are obtained in collaboration with F. Ferrari (University of Bologna) and N. Forcillo (Michigan State University).

**Quantitative Alexandrov theorem and its applications
in the volume preserving mean curvature flow**

Francesca Oronzio

KTH Royal Institute of Technology

☉ June 7, 10:15

A classical theorem in differential geometry, Alexandrov Theorem, states that if Σ is a closed connected embedded smooth surface in \mathbb{R}^n ($n \geq 2$), then it is a round sphere. In this talk, a new quantitative version of it will be given in \mathbb{R}^3 . Using it we obtain a result on the asymptotic of weak solutions for the volume preserving mean curvature flow. Here, by weak solution we mean a flat flow, obtained via the minimizing movement scheme. The results discussed are obtained by a collaboration with V. Julin, M. Morini, and E. Spadaro.

Venue

All talks will take place at the Mathematics Department of the Alma Mater Studiorum Università di Bologna, located in Piazza di Porta S. Donato 5, Bologna.

The room is **Aula Vitali**, on the ground floor.

Social dinner

The social dinner will be held on Thursday June 6 at

DUEBBi L'abruzzese

located in via Mascarella 84b, Bologna. It is at walking distance from the Mathematics Department.

We meet at **20:00** directly in front of the restaurant.

We regret that, for organizational and financial reasons, the dinner is **reimbursed for speakers only**. Registered participants who have indicated in the registration form their wish to participate should cover the cost for themselves.

Programme

	Thursday 6 June	Friday 7 June
09:30 – 10:15		Merlino
10:15 – 11:00		Oronzio
11:00 – 11:30	<i>welcome</i>	<i>break</i>
11:30 – 12:15	Corni	Durastanti
12:15 – 13:00	Cosenza	Franceschi
13:00 – 14:30	<i>lunch break</i>	
14:30 – 15:15	Marini	
15:15 – 16:00	Luzzini	
16:00 – 16:30	<i>break</i>	
16:30 – 17:15	Guarino Lo Bianco	
17:15 – 18:00	Esposito	
20:00	<i>Dinner</i>	