New Frontiers in Parabolic Dynamics and Renormalization

Schedule				
Monday	Tuesday	Wednesday	Thursday	Friday
9:30-10:30 Anton Zorich	9:30-10:30 Davide Ravotti	9-10 Adam Kanigowski	9:30-10:30 Boris Solomyak	9:30-10:30 Pedram Safaee
Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11-12 Minsung Kim	11-12 Kelly Yancey	10:30-11:30 Olga Paris-Romaskevich 11:40-12:40 Francisco Arana-Herrera	11-12 Selim Ghazouani	11-12 Alex Eskin End of the conference
Lunch	Lunch	Lunch	Lunch	
14-15 Carlangelo Liverani	14-15 Wenyu Pan	Free afternoon	14-15 Daren Wei	
Coffee break	Coffee break		Coffee break	
15:30-16:30 Yi Pan	15:30-16:30 Maria Saprykina		15:30-16:30 Or Landesberg	
16:45-17:45 Jon Chaika	16:45-17:45 Artur Avila		16:45-17:45 Federico Rodriguez Hertz	
			20 Banquet Dinner	

Titles and abstracts

1. Adam Kanigowski - **Horocycle orbits at sparse times**
   Abstract: We discuss the orbits of the horocycle flow acting on $SL(2,\mathbb{R})/\Gamma$ when sampled at sparse subsets of the integers. We will focus on the case of polynomial (sub-quadratic) ti- mes and also at numbers which are products of at most two prime numbers. We will show that for every $\delta > 0$, the orbit $\{h_{n^{2-\delta}}x\}$ is dense for every non-periodic point $x \in SL(2,\mathbb{R})/SL(2,\mathbb{Z})$ (joint work with M. Radziwill). We will also show that if Γ is arithmetic, then $\{h_{p\cdot q}x\}$ p,q-primes equidistributes towards the only invariant measure on the regular orbit when sampled over $\mathbb{Z}$ (joint with G. Forni, M. Lemanczyk, M. Radziwill).

2. Alex Eskin - **Measure classification and orbit closures on products of moduli spaces**
   Abstract: We prove the same rigidity theorems for the diagonal SL(2,R) action on products of moduli spaces that were previously proved for the action on a single moduli space. A major part of the proof is derived from a more general theorem in smooth dynamics. I will state that theorem, and indicate how to apply it to the moduli space setting.

3. Anton Zorich - **Random square-tiled surfaces and random multicurves in large genus**
   Abstract: (after joint works with V. Delecroix, E. Goujard and P. Zograf) Moduli spaces of Riemann surfaces and related moduli spaces of quadratic differentials are parameterized by a genus g of the surface. Considering all associated hyperbolic (respectively flat) metrics at once, one observes more and more sophisticated diversity of geometric properties when genus grows. However, most of metrics, on the contrary, progressively share certain rules. Here the notion of "most of" has explicit quantitative meaning, for example, in terms of the Weil-Petersson measure. I will present some of these recently discovered large genus universality phenomena.
   I will use count of metric ribbon graphs (after Kontsevich and Norbury) to express Masur-Veech volumes of moduli space of quadratic differentials through Witten-Kontsevich correlators. Then I will present Mirzakhani's count of simple closed geodesics on hyperbolic surfaces. We will proceed with description of random geodesic multicurves and of random square-tiled surfaces in large genus. I will conclude with a beautiful universal asymptotic formula for the Witten-Kontsevich correlators predicted by Delecroix, Goujard, Zograf and myself and recently proved by Amol Aggarwal.

4. Artur Avila - **tba**
   Abstract: tba

5. Boris Solomyak - **Local spectral estimates and quantitative weak mixing for substitution Z-actions**

Abstract: Substitutions serve as models and test cases for many classes of parabolic systems; in particular, they include codings of self-similar interval exchange transformations. We study (local) spectral properties and related questions of quantitative weak mixing for substitution Z-actions, building on our earlier work on suspensions and translation flows. In particular, we show that in the case when the substitution matrix is irreducible over the rationals and has no eigenvalues of modulus one, weak mixing implies that the spectral measures of "nice" test functions are (at least) uniformly log-Hoelder regular, and hence the Z-action admits power-logarithmic bounds for the rate of weak mixing. On the other hand, for substitutions of Salem type, the spectral measures are shown to have Hoelder upper bounds at the spectral parameters from the number field generated by the Salem number, but the Hoelder exponent at those parameters cannot be chosen uniformly. This is a joint with A. I. Bufetov and J. Marshall-Maldonaldo.

6. Carlangelo Liverani - **Heat equation from a deterministic dynamics**
   Abstract: I'll describe a derivation of the heat equation in the thermodynamics limit, with a diffusive scaling, from purely deterministic dynamics satisfying Newton's equations under a time-dependent external field. (Work in collaboration with G.Canestrari and S. Olla)

7. Daren Wei - **Time change rigidity for unipotent flows**
   Abstract: Two flows are said to be Kakutani equivalent if one is isomorphic to the other after time change, or equivalently if there are Poincare sections for the flows so that the respective induced maps are isomorphic to each other. Ratner showed that if G=SL(2,R) and Γ is a lattice in G, and if U is a one parameter unipotent subgroup in G then the U action on G equipped with Haar measure is loosely Bernoulli, i.e. Kakutani equivalent to a circle rotation. Thus any two such systems SL(2,R)/Γ are Kakutani equivalent to each other. On the other hand, Ratner showed that if G'=SL(2,R) x SL(2,R) and Γ' is a reducible lattice, and U' is the diagonally embedded one parameter unipotent subgroup in G', then (G'/Γ', U', m) is not loosely Bernoulli. We show that in fact in this case and many other situations one cannot have Kakutani equivalence between such systems unless they are actually isomorphic. This is a joint work with Elon Lindenstrauss.

8. Davide Ravotti - **Abelian covers of horocycle flows**
   Abstract: In this talk, we consider the unit speed horocycle flow on infinite Abelian covers of compact surfaces of negative curvature. We discuss a result, joint with Roberto Castorrini, which describes the asymptotics of ergodic integrals for sufficiently regular observables. In the case of hyperbolic surfaces, we recover a result by Ledrappier and Sarig, for which we additionally provide explicit error rates.

9. Federico Rodriguez Hertz - **Rigidity for conjugacies of Anosov flows in dimension 3**

Abstract: In this ongoing project with A. Gogolev and M. Leguil we show that if two transitive Anosov flows in dimension are conjugated (i.e. time preserving orbit equivalence) and the Jacobian of return times match then the conjugacy is actually smooth. This allows us to derive some corollaries about smoothing systems. In this talk I plan to discuss these results and related problems.

10. Francisco Arana-Herrera - **Closed geodesics on surfaces: topology, geometry, arithmetic**
Abstract: Closed geodesics on surfaces can be studied from different points of view: topology, geometry, arithmetic. In this talk we discuss the question of how homologically complicated are long simple closed geodesics on hyperbolic surfaces. The approach of the discussion is via Teichmuller dynamics. This talk encompasses joint work with Giovanni Forni and joint work in progress with Pouya Honaryar.

11. Jon Chaika - **Some results on weak mixing**
Abstract: Avila-Forni proved that the vertical flow on almost every translation surface is weakly mixing. This talk will begin with the key criterion to rule out eigenvalues, the Veech criterion. From there it will address the two main results of this talk:
1) That there is a weakly mixing billiard (in a necessarily irrational) polygon and
2) The flow in almost every direction on a fixed translation surface is weakly mixing, unless the translation surface has a specific obstruction. In the presence of this obstruction the translation flow is not weakly mixing in any direction.

This is joint work with Giovanni Forni and Francisco Arana Hererra and Giovanni Forni.

12. Kelly Yancey - **From IETs to Rank-1 Maps**
Abstract: In this talk I outline my mathematical journey since working with Giovanni Forni as a postdoc at the University of Maryland. I begin by discussing interval exchange transformations, specifically dynamical properties of the self-similar variety. The general theme of the talk is the incompatibility of rigidity and weakly mixing in certain cases, even though the generic map satisfies both. I end with an overview of work with Jon Fickenscher on bounding the partial rigidity of rank-1 maps with a bounded number of cuts and bounded spacer sequence. I will outline the proof and provide examples that show our bound is sharp.

13. Maria Saprykina - **KAM-rigidity for parabolic affine abelian actions on the torus**
Abstract: Two famous instances of local rigidity for $\mathbb{Z}^2$-actions are the classical KAM rigidity of Diophantine toral translations and smooth rigidity of hyperbolic or partially hyperbolic higher rank actions proved by Damjanovic and Katok. To complete the study of local rigidity of affine $\mathbb{Z}^2$-actions on the torus, we address the case of parabolic affine actions.

Consider an affine $\mathbb{Z}^2$-action (a,b) on $\mathbb{T}^d$ generated by two commuting parabolic affine maps of the form $a(x) = A(x) + \alpha$, $b(x) = B(x) + \beta$, where $A, B \in SL(d,\mathbb{Z})$.

We say that the action (a,b) is *KAM-rigid under volume-preserving perturbations* if there exists $\sigma \in \mathbb{N}$, $r_0 \geq \sigma$ and $\varepsilon > 0$ satisfying the following. If $r \geq r_0$ and $(F,G) = (a+f, b+g)$ is a smooth λ-preserving $\mathbb{Z}^2$-action such that

$$\|f\|_r \leq \varepsilon, \quad \|g\|_r \leq \varepsilon, \quad \widehat{f} := \int_{\mathbb{T}^d} f d\lambda = 0, \quad \widehat{g} := \int_{\mathbb{T}^d} g d\lambda = 0,$$

then there exists $H = \mathrm{Id} + h \in \mathrm{Diff}^\infty_\lambda(\mathbb{T}^d)$ such that $\|h\|_{r-\sigma} \leq \varepsilon$ and

$$H \circ (a+f) \circ H^{-1} = a, \quad H \circ (b+g) \circ H^{-1} = b.$$

Let $\mathcal{T}(A,B)$ denote the set of possible translation parts (α,β) in the affine actions with linear part (A,B), that is $\mathcal{T}(A,B) := \{\alpha,\beta \in \mathbb{R}^d \mid (A-\mathrm{Id})\beta = (B-\mathrm{Id})\alpha\}$.

We present the following dichotomy for a commuting pair (A,B) of parabolic matrices, where A is step-2 (i.e., $(A-Id)^2 = 0$):

(*i*) either for any choice of $(\alpha,\beta) \in \mathcal{T}(A,B)$, the affine action (a,b) has a rank one factor that is different from a nonzero translation, in which case the action is not KAM-rigid,

(*ii*) or for almost every choice of $(\alpha,\beta) \in \mathcal{T}(A,B)$ the action (a,b) is ergodic and KAM-rigid under volume preserving perturbations.

This is the result of a joint work with D. Damjanovic and B. Fayad.

14. Minsung Kim - **Cohomological equations for locally Hamiltonian flows**

Abstract: The cohomological equation is a functional equation that appears in various aspects of dynamical systems. Cohomological equations for area-preserving flows were solved by G. Forni, and they were applied to prove the deviation of ergodic integrals for the flow. In this talk, we study the cohomological equation for smooth locally Hamiltonian flows on a minimal component of compact surfaces. I will introduce new obstructions to solving the equations (so-called invariant distributions) that did not appear in the previous settings.

Our main theorem states that the regularity of the solution depends not only on the vanishing of Forni's invariant distributions. It also depends on vanishing a family of new invariant distributions, reflecting the behavior around the singularities (saddles). Our result provides a complete solution to the regularity problem for almost all locally Hamiltonian flows. This is joint work with Krzysztof Frączek.

15. Olga Paris-Romaskevich - **From tiling billiards to Novikov's conjecture**
Abstract: A tiling billiard is a model for the light propagation in heterogeneous media : a ray of light moves through a tiling of a plane, and refracts each time it crosses a boundary of a tile. Fixing the refractive index to be equal to -1 helps to (sometimes) reduce the study of such dynamics to the 1-dimensional dynamics, that of interval exchange transformations with flips.

    Sergei Novikov, inspired by conductivity physics, asked the following topological question. Consider a 3-periodic surface S in the Euclidian space. Take a unit vector n and consider the family of planes P_n orthogonal to it. How often do the connected components of intersections of S with planes from P_n wander to infinity in a non-linear (or « chaotic ») way ? Novikov's conjecture states that the set of corresponding normal vectors n is « small » — it has Hausdorff dimension smaller than 2.

    We show that Novikov's problem in the case of centrally symmetric surfaces of genus 3 (the most interesting case from the point of view of physics) has a reformulation in terms of tiling billiards. This interpretation helps us to give a partial solution to Novikov's conjecture — for generic surfaces, the set of chaotic directions has measure 0.

    The talk is based on a series of works, in collaboration with Ivan Dynnikov, Pascal Hubert, Paul Mercat and Alexandra Skripchenko.

16. Or Landesberg - **(Non-)Rigidity of Horocycle Orbit Closures in $\mathbb{Z}$-covers**
Abstract: tba

17. Pedram Safaee - **Quantitative weak mixing for IETs**
Abstract: We will start by introducing interval exchange transformations and mention some of their ergodic theoretical features. Then, we will state our main results concerning the speed of the decay of Cesaro average of correlations of sufficiently regular functions. The rates of decay distinguish the case of IETs obtained by returns of translation flows to transverse sections on Tori and those of higher genus surfaces. We will then discuss some ideas of the proofs. This is based on joint work with Giovanni Forni and Artur Avila.

18. Selim Ghazouani - **Horocycle averages and quantum chaos**
Abstract: I will start by reminding the audience of the classical connection between the Schrödinger equation and the geodesic flow on a Riemannian manifold, and the breakdown of this approximation due to interference effects at the so-called Ehrenfest time. I will then put forward a (partially conjectural) connection between interference effects in the quantum world and ergodic averages of horocycle flows, when the dynamics of the geodesic flow is Anosov.

19. Wenyu Pan - **On the dimension of limit sets on the real projective plane via stationary measures**
Abstract: We consider the (semi)group action of SL(3, R) on RP^2 a prime example of non-conformal, non-linear, and non-strictly contracting action. We study the Hausdorff dimension of a dynamically defined limit set in RP^2 and generalize the classical Patterson-Sullivan formula. A prominent example is Anosov representations in SL(3, R), for which we prove the equality between the Hausdorff dimensions and the affinity exponents of their limit sets. As an application, it reveals a dimension jump in the Barbot component, which is a local generalization of Bowen's dimension rigidity result. Another example is the Rauzy gasket, for which we confirm a folklore conjecture about the Hausdorff dimension of the gasket. These results originate from a dimension formula of stationary measures on RP^2. This talk is based on the joint works with Yuxiang Jiao, Jialun Li, Disheng Xu.

20. Yi Pan - **Reducibility of quasi-periodic symplectic cocycles**
Abstract: Reducibility of quasi-periodic symplectic cocyles is related to the spectrum of discrete Schrödinger operators on strips. We will talk about a global reducibility result: given one parameter family of such analytic cocycles, for almost every parameter, either the maximal Lyapunov exponent is positive, or the cocycle is analytically (almost) conjugate to some precise model. The techniques include Kotani thoery, KAM theory and in particular study of hyperbolicity of renormalization operator. This is a joint work with Artur Avila and Raphaël Krikorian.