From tiling billiards to Novikov's conjecture

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Abstract: A tiling billiard is a model for the light propagation in heterogeneous media : a ray of light moves through a tiling of a plane, and refracts each time it crosses a boundary of a tile. Fixing the refractive index to be equal to -1 helps to (sometimes) reduce the study of such dynamics to the 1-dimensional dynamics, that of interval exchange transformations with flips.

Sergei Novikov, inspired by conductivity physics, asked the following topological question. Consider a 3-periodic surface S in the Euclidian space. Take a unit vector n and consider the family of planes P_n orthogonal to it. How often do the connected components of intersections of S with planes from P_n wander to infinity in a non-linear (or « chaotic ») way ? Novikov's conjecture states that the set of corresponding normal vectors n is « small » — it has Hausdorff dimension smaller than 2.

We show that Novikov's problem in the case of centrally symmetric surfaces of genus 3 (the most interesting case from the point of view of physics) has a reformulation in terms of tiling billiards. This interpretation helps us to give a partial solution to Novikov's conjecture — for generic surfaces, the set of chaotic directions has measure 0.

The talk is based on a series of works, in collaboration with Ivan Dynnikov, Pascal Hubert, Paul Mercat and Alexandra Skripchenko.