(Non-)Rigidity of Horocycle Orbit Closures in Z-covers

Or Landesberg

Abstract: Horospherical group actions on homogeneous spaces are famously known to be extremely rigid. In finite volume homogeneous spaces, it is a special case of Ratner's theorems that all horospherical orbit closures are homogeneous. Rigidity further extends in rank-one to infinite volume but geometrically finite spaces. The geometrically infinite setting is far less understood. In this talk, we'll consider $\mathbb{Z}$-covers of compact hyperbolic surfaces and describe their horocycle orbit closures. We expose a connection between the structure of such orbit closures and the topology of an associated geodesic lamination on the compact surface, a lamination which delicately depends on the choice of a metric. These results show, in particular, that horocycle orbit closure rigidity breaks down in the geometrically infinite context. Nonetheless, in the setting of $\mathbb{Z}$-covers, some rigidity is preserved in the form of integer Hausdorff dimensions=1,2, or 3. Based on joint work with James Farre and Yair Minsky.