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Invited conferences

From spline error estimates to outlier-free isogeometric discretizations

Carla Manni
University of Rome “Tor Vergata”
manni@mat.uniroma2.it

joint work with Espen Sande and Hendrik Speleers

Isogeometric analysis is a well established paradigm to improve interoperability between geometric modeling and numerical simulations. It is commonly based on B-splines, and their rational generalization (NURBS), and shows important advantages over classical C^0 finite element analysis. In particular, the inherently high smoothness of B-splines/NURBS allows for a higher accuracy per degree of freedom. This has been numerically observed in a wide range of applications, and recently a mathematical explanation has been given thanks to error estimates in spline spaces with constants that are explicit in the polynomial degree p and the smoothness k.

The isogeometric approach based on maximally smooth spline spaces over uniform grids is an excellent choice for addressing eigenvalue problems. Yet, it still presents a flaw: a very small portion of the eigenvalues are poorly approximated and the corresponding computed values are much larger than the exact ones. These spurious values are usually referred to as outliers. On the other hand, outlier-free discretizations are appealing in various contexts.

In this talk we review some recent results on a priori error estimates with explicit constants for approximation by splines of arbitrary smoothness on general knot sequences, [1,2]. These estimates are sharp or very close to sharp in several interesting cases and are actually good enough to cover convergence to eigenfunctions of classical differential operators under k-refinement. The application to outlier-free discretizations is also discussed, [3].

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Rational spline manifolds

Hartmut Prautzsch
Karlsruhe Institute of Technology
prautzsch@kit.edu

Building smooth freeform surfaces requires reparametrizations that if one works with piecewise polynomial surfaces - and this includes subdivision surfaces - typically lead to much higher degrees than for equally smooth spline functions over just planar domains.

Low degrees are possible though if one works with rational splines using ideas that have been introduced to CAGD already in 1993. This has been further investigated only in a few articles, see e.g. [1,2]. Splines built according to these ideas are reparametrized over the hyperbolic plane and have G^k joints with linear rational transition functions forming what is called a projective structure. This allows to represent these so-called orbifold splines as trivariate piecewise polynomial homogeneous splines with simple C^k joints and to use, e.g., B-patches or Powell-Sabin split constructions to generate surfaces of minimal degree.

However, the dependence on fundamental domains in the hyperbolic plane causes certain difficulties with the consequence that these ideas have not found widespread use so far. For example, the construction of orbifold splines is amenable particularly to triangular patches while working with quadrilateral or even multisided patches has not or only recently been investigated. Further it is not known, if orbifold splines can help to develop good subdivision algorithms.

In this talk I will review the construction of orbifold splines, address current difficulties, show how to easily obtain projective structures from any mesh coarsely representing a desired surface and its patch layout, and present constructions with T-junctions, quadrilateral and multisided "S"-patches.

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Mathematical imaging: from geometric PDEs and variational modelling to deep learning for images

Carola-Bibiane Schönlieb
Centre for Mathematical Sciences, Cambridge, United Kingdom
cbs31@cam.ac.uk

Images are a rich source of beautiful mathematical formalism and analysis. Associated mathematical problems arise in functional and non-smooth analysis, the theory and numerical analysis of nonlinear partial differential equations, inverse problems, harmonic, stochastic and statistical analysis, and optimisation. In this talk we will learn about some of these mathematical problems, about variational models and PDEs for image analysis and inverse imaging problems as well as recent advances where such mathematical models are complemented and replaced by deep neural networks. The talk is furnished with applications to art restoration, forest conservation and cancer research.

Approximate C^1-smoothness in isogeometric analysis

Thomas Takacs
Johann Radon Institute for Computational and Applied Mathematics (RICAM)
thomas.takacs@ricam.oeaw.ac.at

joint work with Deepesh Toshniwal, Pascal Weinüller

Isogeometric analysis (IGA) is a numerical method based on B-spline or NURBS representations of geometries. One of the main advantages of IGA is its use of smooth splines. Isogeometric discretizations based on tensor-product B-splines or NURBS of degree p may be up to C^{p-1}-smooth globally. Thus, such spline spaces can be used to discretize higher order partial differential equations (PDEs) using a Galerkin approach. In this talk we particularly focus on C^1-smooth spline constructions over multi-patch domains, cf. [1], where higher order smoothness becomes non-trivial. Such constructions, as summarized in [2], can be used to solve fourth order PDEs.

The goal is to obtain an isogeometric discretization which possesses good approximation properties while satisfying the necessary smoothness requirements. It turns out that for many multi-patch geometries C^1-smoothness conditions are too restrictive, preventing convergence to the correct solution. In order to circumvent this issue, one may modify the space by locally relaxing the smoothness requirements and/or enlarging the spline space, e.g., as in [3,4]. We study the effects of such local modifications and discuss how one can construct (approximately) C^1-smooth, non-nested discretization spaces, which are well-suited for isogeometric analysis of fourth order PDEs.

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Variational learning with simplicial splines

Michael Unser
Biomedical Imaging Group, EPFL, Lausanne, Switzerland
`Michael.Unser@epfl.ch`

joint work with Alexis Goujon, Mehrsa Pourya, Shayan Aziznejad

It is well known that deep ReLU networks produce regression maps that are continuous and piecewise linear (CPWL); in other words, these are high-dimensional linear splines. As alternative, we investigate a functional-optimization framework that promotes simple CPWL solutions (e.g., with a minimal number of faces). We do so by imposing a penalty on the Hessian total variation (HTV) [1]. Our formulation lends itself to an exact discretization with the help of simplicial splines [2], including box splines [3]. In particular, we show that the simplicial spline representation is stable—by providing Riesz bounds—and that it admits an explicit formula for the HTV criterion. This then naturally leads to the design of an efficient algorithm for the learning of CPWL regressors from data. We discuss the properties of our scheme and present experimental comparisons with kernel methods and neural networks.

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Geometric subdivision and multiresolution

Johannes Wallner
TU Graz
j.wallner@tugraz.at

This survey talk discusses geometric operations on data, in particular refinement operations used in subdivision and multiscale analysis. If data do not live in an affine space, linearity of operations ceases to be meaningful and has to be replaced by the property that operations respect the intrinsic geometry of the data. This applies already to the well-studied case of averages. As to subdivision processes, the technical difficulties dealing with nonlinear operations have successfully been mastered with regard to the question of smoothness of limits (even in the combinatorially irregular case). Likewise a limited choice of multiscale transforms is available and has been analyzed. However, contrary to first expectations, the actual convergence of refinement processes has proved most resistant to analysis. Satisfactory results have been obtained for special subdivison rules, namely interpolatory ones, and for special geometries, namely for CAT(0) metric spaces. Here, a sharp result characterizing convergence has been achieved for subdivision schemes with nonnegative mask by [1]. For smooth manifolds, in the more general setting of nonpositive curvature and no restrictions on the sign of a subdivision rule's coefficients, a sufficient condition guaranteeing convergence has been derived by [2]. It is analogous to the conditions known from the linear case. As to spaces with positive curvature, sensible density bounds on input data guaranteeing convergence have so far been derived only for highly symmetric geometries [3].

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Contributed talks

A nonlinear B-spline quasi-interpolation method

Francesc Aràndiga
Universitat de València, Spain
arandiga@uv.es

joint work with R. Donat, S. López-Ureña

Quasi-Interpolation based on B-spline approximation methods are used in numerous applications. However, we observe the Gibbs phenomenon that is obtained when approximating near discontinuities. We present a nonlinear modifications of the method, based on weighted essentially non-oscillatory (WENO) techniques, to avoid these phenomena near discontinuities and, at the same time, obtain high-order accuracy in smooth regions.

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[2/2] **Rational non-stationary Hermite interpolatory subdivision scheme**

Shubhashree Bebarta
Veer Surendra Sai University of Technology
Burla Sambalpur, Odisha, India
shubhashreebebarta2009@gmail.com

joint work with Mahendra Kumar Jena

A lot of studies have been done on the development of curves and surfaces based on subdivision schemes throughout the previous four decades. Non-stationary subdivision schemes, on the other hand, have gotten a lot of interest due to their usefulness in geometric modeling and Computer Aided Geometric Design (CAGD). We suggest a new subdivision method for the Rational Trigonometric Bernstein Bézier (TBB) Spline in this article. The scheme is known as the Hermite interpolatory subdivision scheme because the subdivision scheme can only be obtained through Hermite interpolation. The subdivision scheme's regularity (convergence, smoothness) and shape-preserving (monotonicity, convexity) features are also investigated.

Multi-degree B-splines and their stable evaluation

Carolina Beccari
Alma Mater Studiorum - Università di Bologna
carolina.beccari2@unibo.it

joint work with Giulio Casciola

Multi-degree splines are piecewise functions constructed by gluing together finitely many polynomial pieces of (possibly) different degrees with some specified orders of smoothness. As such, they are a natural extension of the standard "uniform-degree" polynomial splines, with the advantage that they generally allow for modeling complex geometries with fewer coefficients. This feature has made them of interest in various contexts, ranging from geometric design to engineering analysis and image processing. The largest class of such splines, which is the one we are dealing with, allows continuity up to C^r, $r = \min\{d_i, d_{i+1}\}$, between two subsequent pieces having different degree d_i and d_{i+1}. For these spaces, basis functions having similar properties as standard B-splines (dubbed multi-degree B-splines) can be defined on suitable knot partitions by means of integral recurrence relations [1]. The existence of such bases is a prerequisite in many applications; among these, we will illustrate some examples in the modeling of geometric shapes.

We will then present a practical method for the evaluation of multi-degree B-splines, which consists in constructing a linear operator that maps the multi-degree B-spline basis into a set of local B-spline functions, connected with C^0 continuity where the degree changes [2]. The latter can be evaluated locally using the stable algebraic recurrence formula for standard B-splines. Moreover, the linear operator relating the two bases is computed in a numerically stable manner, as confirmed by both theoretical arguments and numerical evidence.

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Discontinuity detection-based meshless numerical method for conservation laws

Cesare Bracco
Department of Mathematics and Computer Science
University of Florence
cesare.bracco@unifi.it

joint work with Oleg Davydov, Carlotta Giannelli, Alessandra Sestini

Conservation laws are models describing the dynamic of several important physical quantities. Mathematically, they are challenging problems, as discontinuities may appear in the solution even if the initial condition is smooth. The presence of discontinuities is numerically very challenging, since it often leads to an unstable solution if not handled with a suitable approach [3]. For this reason, the method we present is "discontinuity-aware", meaning that, as time advances, we detect and suitably treat the (possible) discontinuities of the solution. The method is based on discretizing in space the equation by approximating the divergence of the flux with constrained numerical differentiation formulas [2]. Since such formulas are designed for scattered data, the method is meshless. At each time step, we use an indicator [1] based on the same differentiation formulas to detect the discontinuity points, where we then add an artificial viscosity term. This correction is often crucial in order to recover the correct shape of the solution. The resulting scheme is monotone, which guarantees the stability and the absence of oscillations. We will present a selection of examples highlighting the features of the method.

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Signal despeckling with learned regularisation

Simone Cammarasana
CNR-IMATI
simone.cammarasana@ge.imati.cnr.it

joint work with Paolo Nicolardi, Giuseppe Patané

The despeckling of arbitrary signal through regularisation is a relevant topic in biomedical applications [1]. In particular, ultrasound (US) signals are affected by speckle noise that significantly affects the evaluation of the morphology of the anatomical district. We propose a novel method for the despeckling of arbitrary signals, which exploits learned regularisation. Given a data set of ground-truth signals, we generate an artificial speckle noise, apply the Singular Values Decomposition, and compute the optimal thresholds for the singular values that allow the best reconstruction of the ground-truth signal. We iterate this approach, where the input signal of each iteration is the despeckled signal at the previous step. The input and optimal parameters compose the training data set, which is used to train a learning-based model to predict the optimal thresholds of the regularisation. Finally, we apply our learning model to despeckle US 2D images, 3D images, and 2D videos.

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An algorithm for non negative P-spline

Rosanna Campagna
University of Campania "L.Vanvitelli", Italy
rosanna.campagna@unicampania.it

Penalized regression splines are largely used to investigate and predict data behaviour. Different penalized models are available in literature and the most commonly used are surely the P-splines [1], based on two main ingredients: a basis of polynomial B-splines and a second order discrete difference penalty. In the applications, the problem requirements generally translate into modelling constraints or first order penalties (e.g. in social and behavioural sciences [2]). Moreover, P-splines are used for Bayesian spectral density estimation; e.g. in [3] the authors propose statistic techniques for a data-driven selection of the spline knots placement, so giving up the uniform distribution of the knots which guarantees analytical P-spline reproduction properties [4]. With the aim to gain the non-negativity of the P-spline model without giving up its analytical properties, we define the Non Negative P-splines (NNP-spline) that is the solution of a suitable constrained optimization problem. The new model is defined to capture the data trend and to result *almost everywhere* non negative in the data domain, while preserving the constant spacing of the knots. We propose a greedy-type algorithm for selecting additional points, based on a local first order approximation. A theoretical result relating the boundaries on the constraints to variational properties of the B-splines is also investigated and drives the selection.

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Accelerating some algorithms for CAGD and dual Bernstein bases

Filip Chudy
Institute of Computer Science
University of Wrocław
fch@cs.uni.wroc.pl

joint work with Paweł Woźny

The main aim of the talk is to briefly present the content of the PhD thesis [5]. Some results are new, while some have already been published in [1–4]. The proposed methods allow to accelerate the computations performed, e.g., in computer graphics, approximation theory, and numerical analysis.

For example, an algorithm for fast evaluation of a Bézier curve combines the linear complexity of the Horner's scheme and the geometric and numerical advantages of the de Casteljau algorithm. The new method can be generalized to evaluate, e.g., rational Bézier surfaces with optimal complexity.

A new differential-recurrence relation satisfied by B-spline functions allows to compute their Bernstein coefficients in linear time. Together with the new method for evaluating a Bézier curve, one can evaluate many B-spline curves at multiple points faster than when using the de Boor-Cox algorithm.

Recurrence relations satisfied by dual Bernstein polynomials of the same degree are given. They allow to find the values of all such polynomials in optimal time. This result can be used to reduce the cost of using dual projections which is very useful in, e.g., CAGD and numerical analysis.

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3D IgA-BEM with nonconformal C^0 multipatch spline spaces

Bruno Degli Esposti
University of Florence
bruno.degliesposti@unifi.it

joint work with Tadej Kanduč, Alessandra Sestini

Isogeometric Analysis (IgA) can be combined with Boundary Element Methods (BEM) to yield numerical schemes that solve PDEs on volumetric domains using exclusively their multipatch NURBS boundary representation, as generated by CAD software. Globally C^0 discretization spaces are usually obtained by identifying corresponding B-splines degrees of freedom on opposite sides of patch interfaces under the assumption of *conformity*, i.e. that the 1D spline knot vectors T_L, T_R coincide on the two sides of each edge ($T_L = T_R$). Unfortunately, this constraint propagates globally across patches, reducing the flexibility of the approach.

In this talk we propose a generalization to the *nonconformal* setting of the C^0 multipatch IgA-BEM collocation method considered in [1] for the numerical solution of 3D Helmholtz problems. We show that convergence and C^0 regularity in the case $T_L \neq T_R$ can still be attained if the cardinality $|T_L \cap T_R|$ grows proportionally to $|T_L|$ and $|T_R|$. Our approach is based on *not-a-knot* conditions and does not require master-slave relations between neighbouring patches. The benefits of nonconformal schemes will be presented for suitable IgA-BEM applications, exploiting the possibility of choosing spline spaces on each patch in an essentially independent way.

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Smooth approximation and interpolation of scattered data on the sphere with linear precision by quadrangulations

Francesco Dell'Accio
University of Calabria - Department of Mathematics and Computer Science
francesco.dellaccio@unical.it

joint work with Filomena Di Tommaso, Federico Nudo

The problem of interpolation on the sphere arises in the study of many physical phenomena, such as temperature, rainfall, pressure, ozone distribution or gravitational forces, measured at scattered points on the earth surface. A comprehensive survey on various approaches to solve this problem has been provided by G.E. Fasshauer and L.L. Schumaker in 1998 [5]. Later, in 2010, R. Cavoretto and A. De Rossi proposed an approach which involves a modified spherical Shepard's interpolant and zonal basis functions as local approximants [1]. In line with previous studies [2,3,4], in this talk we discuss a new approach based on quadrangular Shepard basis functions on the sphere combined with linear interpolants at quadrangulations of the scattered points. In particular, the basis functions are the normalization of the product of the inverse geodesic distance to the vertices of the quadrangulation while the linear interpolants are defined by spherical polynomials [6]. The resulting operator reproduces linear polynomials on the sphere and interpolates the given data. Numerical experiments on variuos sets of scattered points demonstrate the effectiveness of the approximation.

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Hierarchical matrices techniques for Helmholtz problem in IgABEM setting

Giuseppe Alessio D'Inverno
University of Siena
dinverno@diism.unisi.it

joint work with Maria Lucia Sampoli, Alessandra Sestini

Non-sparsity of system matrices obtained by Boundary Element Methods is a well-known issue, which requires approximation techniques to reduce storage and computational complexity as the dimension N of the system increases [1]. With the Isogeometric Analysis (IgA-BEM) setting, the sizes of the matrix are reduced, yet they are still dense and need to be handled with the aforementioned techniques. In this talk, hierarchical matrices theory [2] is considered to achieve simpler linear systems and the first results are presented when this approach is applied to solve numerically the Helmholtz problem on a spherical domain, which admits a smooth conformal multi-patch representation of the boundary surface. Since the symmetry of the domain allows to easily evaluate the indices partition used to build the hierarchical blocks in the matrix, the overall system solving process is improved in terms of computational efficiency, whereas the error approximation is not significantly affected. Numerical examples are given to compare the method effectiveness at different refinements levels.

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2D prediction operators based on multiquadric local interpolation with adaptive parameter estimation. Applications to image compression

Rosa Donat
Universitat de València, Spain
donat@uv.es

joint work with F. Aràndiga, D. Schenone

We present and analyze three different prediction operators in the 2D setting based on multiquadric radial basis function interpolation with either linear or WENO shape parameter approximation. When considered within Harten's framework for Multiresolution, these prediction operators give rise to sparse multi-scale representations of 2D signals, whose compression capabilities are demonstrated through numerical experiments.

It is well know that the accuracy of multiquadric interpolation depends on the choice of the shape parameter. In addition, in [1], it was shown that the use of data-dependent strategies in the selection of the shape parameter lead to more accurate reconstructions. We shall show that our local adaptive estimates of the shape parameter result in an improvement in approximation power in the 2D framework, with respect to a pure tensor product extension of the 1D operators considered in [2]-[3], where linear estimations of the shape parameter are considered, or [1], where 1D nonlinear parameter estimates are considered.

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Image reconstruction from blind ptychographical measurements

Frank Filbir
Helmholtz Center Munich & Technische Universität München
filbir@helmholtz-muenchen.de

Ptychography is a computational imaging technique. A detector (CCD camera) measures the intensity of many diffraction patterns each obtained by illuminating a small part of the object at a time. The measurements are produced by using light (X-rays) of one specific very short wavelength λ and the detector placed in the far-field distance (Fraunhofer diffraction). Mathematically this experimental set-up leads to a phase retrieval problem from spectrogram measurements, i.e. we are given samples of

$$\mathfrak{I}_\lambda(x,\xi) = \Big| \int_{\mathbb{R}^2} f(y)\, g_\lambda(y-x)\; e^{-2\pi i \xi\cdot y/\lambda}\, dx \Big|^2$$

for a known window function g and we have to reconstruct f. However, there are experimental set-ups which do not allow to work with one specific wavelength λ but we have to deal with polychromatic measurements, i.e. we are given $\mathfrak{I}_\lambda(x,\xi)$ for $\lambda \in \{\lambda_1,\ldots,\lambda_L\}$. Moreover, in many cases even the g_λ's are unknown. This then leads to what is called Blind Polychromatic Ptychographic Imaging (BPPI). In this talk we will provide an overview of BPPI and we present some reconstruction methods and results.

The talk is based on joint work with Oleh Melnyk (Helmholtz Munich), Felix Krahmer (Technische Universität München) and our project partners Jan Rothardt (GSI Jena) and Nico Hoffmann (HZDR, Dresden) within the Helmholtz Imaging Platform project AsoftXm.

Spectral analysis of matrices from isogeometric immersed methods

Carlo Garoni
University of Rome "Tor Vergata"
garoni@mat.uniroma2.it

joint work with Carla Manni, Francesca Pelosi, Hendrik Speleers

We consider the tensor-product B-spline isogeometric analysis discretization of a variable-coefficient symmetric elliptic problem. The isogeometric discretization is coupled with an immersed boundary (embedded domain) method that preserves the symmetry of the problem [1, 5].

We present a spectral analysis of the matrices resulting from this discretization [3]. In particular, our interest is focused on the asymptotic distribution of the eigenvalues as the mesh-fineness parameter n tends to ∞, i.e., as the mesh is progressively refined to get increasingly accurate approximations. Such analysis plays a role both in the design of efficient solvers for the resulting linear systems and in the study of the accuracy with which the proposed discretization method approximates the spectrum of the differential operator underlying the considered elliptic problem.

The spectral analysis tools we use are entirely based on the theory of (reduced) generalized locally Toeplitz (GLT) sequences [2, 4], which is introduced in the talk along with the obtained spectral results.

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On constructing non-negative edge basis functions for representation of splines over triangulations

Jan Grošelj
University of Ljubljana, Slovenia
jan.groselj@fmf.uni-lj.si

joint work with Marjeta Knez

C^1 continuous polynomial splines over general triangulations are commonly characterized by interpolation problems, which are described by functionals related to the vertices, edges, and triangles of the triangulation. The splines are then naturally represented as linear combinations of the basis functions dual to the interpolation functionals.

In recent years, attempts have been made to improve standard representation of splines over triangulations by constructing non-negative basis functions that form a partition of unity. The key idea behind this research is based on a geometric consideration of degrees of freedom related to the vertices of the triangulation, which enables a B-spline-like representation of several types of splines, e.g., Powell–Sabin splines, reduced Clough–Tocher splines, and reduced Argyris splines. However, this approach does not cover the degrees of freedom related to the edges of the triangulation.

In this talk we will present a new approach to capture the degrees of freedom of splines that are related to the edges of the triangulation. For a spline in the Bernstein–Bezier representation defined over two triangles with a common edge, we will show how to replace certain Bernstein basis polynomials by a suitable number of non-negative spline basis functions in a way that C^1 continuity conditions are automatically incorporated into the representation without any additional constraints on the spline coefficients. We will then use this new construction in combination with the well-established construction of vertex basis functions to introduce normalized representations of higher degree Argyris [1] and Clough–Tocher splines [2].

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On spline weighted least square approximation

Sofia Imperatore
University of Florence
sofia.imperatore@unifi.it

joint work with Carlotta Giannelli, Lisa Maria Kreusser, Estefania Loayza-Romero, Fatemeh Mohammadi, Nelly Villamizar

Spline interpolation and least square approximation schemes are widely used in a variety of applications to reconstruct a geometric model from its observations. Even if they are usually addressed as alternative of each other, a reformulation of polynomial least square approximations in terms of a convex combination of polynomial interpolants was recently derived [1]. After extending this result to the spline setting, we introduce a truncated version of the considered spline approximation based on a suitable selection of the weights. Different numerical examples will be presented.

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Bivariate non-uniform subdivision schemes based on L-systems

Ioannis Ivrissimtzis
Department of Computer Science, University of Durham
ioannis.ivrissimtzis@durham.ac.uk

joint work with Cédric Gérot

The aim of this work is to generalise non-uniform univariate subdivision schemes based on L-systems [1] to the regular, quadrilateral case. Like NURSS [2], the mesh edges are labelled with letters associated with B-spline knot interval lengths. But here, the labels are letters of an L-systems alphabet, and at each step the edges are processed according to the L-system's production rules. As L-systems allow for rules substituting a letter with a single letter word, a step could just relabel an edge, without splitting it, allowing this way for schemes of slower than binary refinement. In particular, this technique allows for schemes with non-rational refinement rates.

Unlike the univariate case, in the bivariate case there is no natural ordering of the mesh edges. To deal with that, our production rules, and the corresponding subdivision masks, should be symmetric enough to make the final result independent of the order by which edges are processed. Moreover, while here we only deal with the regular case, we aim at designing a framework that will adapt to extraordinary vertices, and thus, we require stationarity and locality for the masks that will be applied around the initial vertices throughout the subdivision steps.

With the above considerations, we propose two new schemes. The first, has the slowest refinement rate, equal to the golden ratio. The second, has the smallest number of masks, and it turns out to be a binary-ternary scheme.

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Numerical integration for isogeometric BEM applied to 3D Helmholtz problems on multi-patch domains

Tadej Kanduč
University of Ljubljana
tadej.kanduc@fmf.uni-lj.si

joint work with Antonella Falini, Maria Lucia Sampoli, Alessandra Sestini

A collocation isogeometric Boundary Element Method (BEM) is considered to numerically solve Helmholtz equation on 3D multi-patch domains. The discretization space consists of C^0 continuous basis functions, whose restriction to a patch spans a space of tensor product B-splines, composed with the given patch parameterization.

One of the key novel ingredients is the numerical integration scheme that combines higher order analytical singularity extraction process for the governing singular integrals and a B-spline quasi-interpolation numerical integration for the regular integrals. A special care is needed to correctly address integration across patch interfaces. On numerical examples we demonstrate that the expected orders of convergence for the approximate solutions are achieved with a small number of quadrature nodes.

Construction of spatial Pythagorean-hodograph G^2 Hermite interpolants with prescribed arc lengths

Marjeta Knez
University of Ljubljana, Slovenia
marjetka.knez@fmf.uni-lj.si

joint work with Francesca Pelosi, Maria Lucia Sampoli

Polynomial Pythagorean-hodograph (PH) curves ([1]), characterized by the property that their unit tangent is rational, have many important features for practical applications. Spatial PH curves are especially interesting because their construction from a quaternion preimage curve allows one to equip the curve with a rational orthonormal adapted frame which makes these curves a tool for motion design applications. Furthermore, the property of possessing a polynomial arc length function significantly simplifies the computation of the curve with a prescribed length.

In this talk the problem of constructing spatial G^2 continuous PH spline curves, that interpolate points and frame data, and in addition have the prescribed arc length, is addressed. The interpolation scheme is completely local and can be directly applied for motion design applications. Each spline segment is defined as a PH biarc curve of degree 7 satisfying super–smoothness conditions at the biarc's joint point. The biarc is expressed in a closed form with additional free parameters, where one of them is determined by the length constraint. The selection of the remaining free parameters is suggested, that allows the existence of the solution of the length interpolation equation for any prescribed length and any ratio between norms of boundary tangents. By the proposed automatic procedure for computing the frame and velocity quaternions from the first and second order derivative vectors, the results present a direct generalization of the construction given in [2] from planar to spatial curves. Several numerical examples are provided to illustrate the proposed method and to show its good performance, also when a spline construction in considered.

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Towards an evolutionary approximation of subdivision control meshes

Alexander Komar
Institute of Computer Graphics and Knowledge Visualization
Graz University of Technology
a.komar@cgv.tugraz.at

joint work with Ursula Augsdöfer

Although numerical models may be derived ab initio many CAD models are still derived from real-world objects. That is, a real shape is captured and then converted into a numerical representation using either NURBS or subdivision. This process is referred to as surface reconstruction.

Approaches to find a subdivision control mesh for a given input shape are typically based on analysing the given surface and then manually or semi-automatically fit a control mesh which approximates in the limit the smooth input shape while respecting differential properties of the surface.

The surface reconstruction we present avoids analysis of the input shape. Instead we use a genetic algorithm to optimize a set of typical modelling operations to derive a subdivision control mesh which in the limit approximates the input or target shape. The operations are applied to a simple base object like a cube or torus and are randomly initialized.

The genetic algorithm tries to optimize the set of modelling operations not only with respect to the input shape, but also such that a good control mesh for the target shape is achieved. To achieve this a meaningful fitness functions has to be defined that takes into account the distance of the subdivided test shape to the target shape how many operations are applied and how many vertices the subdivided test shape has in comparison to the target shape. A well suited test shape, with a high fitness value should minimize all three measures.

We present our genetic algorithm including metrics for determining the fitness of a test shape.

Quadratures for Gregory Patches

Jiří Kosinka
University of Groningen
j.kosinka@rug.nl

joint work with Jun Zhou, Pieter Barendrecht, Michael Bartoň

We investigate quadrature rules for Gregory patches [1]. We provide numerical and where possible symbolic quadrature rules for the space of polynomial/rational functions associated with Gregory quads, as well as their derivatives, products, and products of derivatives, i.e., the derived (isogeometric) spaces. This opens up the possibility of incorporating Gregory quads in numerical simulations [2] without having to resort to imprecise quadratures. We also cover preliminary results for the case of Gregory triangles.

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Simplex spline bases for smooth splines on refined triangulations

Tom Lyche
University of Oslo
tom@math.uio.no

joint work with Carla Manni, Hendrik Speleers

Splines on triangulations have widespread applications in many areas, ranging from finite element analysis and physics/engineering applications to computer graphics and entertainment industry. High smoothness spline spaces are often preferred.

When dealing with a general triangulation, to obtain splines of high smoothness in a stable manner sufficiently large degrees have to be considered. An alternative is to use lower-degree macro-elements that subdivide each triangle into a number of subtriangles (or more generally subdomains).

Simplex splines are one of the most elegant generalizations of univariate B-splines to the multivariate setting. They can be interpreted as the density function of a simplex shadow. This geometric construction allows to easily derive properties such as smoothness and recursion, knot insertion and degree elevation formulas.

In this talk, after reviewing the main properties of simplex splines, we consider a family of macro-elements of degree d and maximal smoothness $d-1$ on a triangular region and we discuss the construction of a suitable local representation for the related spline space in terms of simplex splines. In particular, we detail the important cases of C^1 quartic, C^2 cubic and C^3 quartic macro-elements and we discuss several interesting properties, such as local support, linear independence, and nonnegative partition of unity of the provided simplex spline basis.

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Point cloud data fitting via G^1 smooth spline basis functions

Michelangelo Marsala
Inria Sophia Antipolis-Méditerranée Université Côte d'Azur, France
michelangelo.marsala@inria.fr

joint work with Angelos Mantzaflaris, Bernard Mourrain

Geometrically smooth spline functions are piecewise polynomial functions defined on a mesh, that satisfy properties of differentiability across shared edges; their unstructured nature provide them a wide range of application like, for example, the point data fitting. Point cloud fitting is a very important topic especially in CAD theory, which permits in fact to translate CAD data into Bézier representation. In the presentation we consider G^1 splines on quadrangular mesh defined with quadratic glueing data function along shared edges. We will describe shortly their construction, their properties, analyze their space, and provide dimension formula. After having showed how to construct an efficient basis for the considered space, numerical results will be presented to illustrate the quality of the fitting.

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Multiscale representations of manifold-valued data via non-interpolating subdivision schemes

Wael Mattar
Tel Aviv University
waelmattar.95@gmail.com

joint work with Nir Sharon

The close connection between subdivision schemes and wavelets has been studied for decades. One milestone is Donoho's work from the nineties about the direct application of interpolatory subdivision operators as upscaling operators in a pyramid transform [1]. However, it has been established only recently how to use non-interpolatory operators similarly [2].

In this talk, we will briefly survey this hierarchical analysis and introduce the lifting of multiscale pyramid transform for analyzing manifold-valued time series. Then, we describe this construction in detail and present its analytical properties, including stability and coefficient decay. Finally, we numerically demonstrate the results and show the application of our method for denoising and anomaly detection.

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On the matrices in B-spline collocation/Galerkin methods for a kind of fractional differential equation

Mariarosa Mazza
University of Insubria
mariarosa.mazza@uninsubria.it

joint work with Marco Donatelli, Carla Manni, Hendrik Speleers

Fractional derivatives are non-local operators that have been demonstrated to be useful when modeling anomalous diffusion phenomena. In this work, we focus on a certain fractional differential equation discretized by a polynomial B-spline collocation/Galerkin method. For an arbitrary polynomial degree, we show that the resulting coefficient matrices possess a Toeplitz-like structure. As such, we investigate their spectral properties via their symbol, a function which describes the asymptotic spectral distribution of a sequence of matrices, as the size tends to infinity. We prove that, like for second order differential problems, also in this case the given matrices are ill-conditioned both in the low and high frequencies for large degrees, with a mitigated conditioning in the low frequencies and a deterioration in the high ones. As a side result of the symbol computation, we find a new way of expressing the fractional derivative of a cardinal B-spline as inner product of two fractional derivatives of cardinal B-splines. Thanks to this, we are able to recognize that, as for classical diffusion problems, the central parts of the Galerkin and collocation matrices share the same pattern. Finally, we perform a numerical study of the approximation order of both approaches.

Generalized spline quasi-interpolants and applications to numerical analysis

Mohamed-Yassir Nour
Hassan First University of Morocco
University of Lorraine, France
mohamedyassirnour@gmail.com

joint work with Abdellah Lamii, Ahmed Zidna, Driss Sbibih.

In this work, we first establish a general Marsden's identity for Unified and Extended B-spline (UE B-splines or Omega B-splines for short). Then, by using this result, we construct univariate omega spline quasi-interpolants on a bounded interval. For particular values of omega, we refind some already developed quasi-interpolants. As a practical side of these operators, we give some applications to numerical analysis especially quadrature formulas, differentiation and numerical solution of linear Fredholm integral equations. We also study the approximation errors of these operators and we illustrate the theoretical results by numerical examples.

Conforming/non-conforming isogeometric de Rham complex discretization in disk-like domains via polar splines: applications to electromagnetism

Francesco Patrizi
Max Planck Institute for Plasma Physics
francesco.patrizi@ipp.mpg.de

joint work with Martin Campos Pinto, Yaman Güçlü

In this talk we present a discretization of the continuous de Rham complex by means of adequate tensor-product spline spaces sustaining the same cohomological structure, when the underlying physical domain has a disk-like shape. Discretizations preserving such topological invariant of the physical model are commonly exploited in electromagnetics to obtain numerical solutions satisfying important conservation laws at the discrete level. Thereby one avoids spurious behaviors and, on the contrary, improves accuracy and stability of the approximations. The singularity of the parametrization of such physical domains demands the construction of suitable restricted spline spaces, called polar spline spaces, ensuring an acceptable smoothness to set up the discrete complex. In particular we present the sub-complex whose 0-forms are C^1 smooth on the domain. Although it seems natural to adopt polar splines [2] as basis functions, these do not have a tensor-product structure near the pole. This has a profound effect when pursuing high-performance computing: fast Kronecker-based algorithms cannot be used, MPI communication patterns are modified near the pole and new data structures may be needed. Therefore, in order to obtain a discretization capable to scale well with problems of large dimensions, we further present a, so called, conforming/non-conforming (CONGA) [1] variation of the method. In this approach, the approximation is built in the ambient space of the tensor splines, by means of standard projectors which ease the parallelization of the processes, and afterwards such approximation is projected on the polar spline subspace to preserve the conformity of the discretization.

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On Kernel-Target alignment for data-driven approximation

Emma Perracchione
Dipartimento di Scienze Matematiche G.L. Lagrange
Politecnico di Torino
emma.perracchione@polito.it

In this talk we discuss the fact that the interpolation via the so-called Variably Scaled Kernels (VSKs) [1] can be seen as a Tikhonov-like approximation with standard kernels. In view of that we analyze, from both a theoretical and a numerical point of view, the advantage of using VSKs for achieving a better kernel alignment [2] that leads to a target-based kernel basis.

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Weyl transform associated with linear canonical wavelets

Akhilesh Prasad

Indian Institute of Technology (ISM), Dhanbad-826004, India

aprasad@iitism.ac.in

joint work with Amit Kumar

In the present paper, we define the linear canonical wavelets and study the corresponding wavelet transforms along with some useful properties and results for it. Parseval identity, inversion formula for the linear canonical wavelet transform are obtained. Weyl transform on the admissible linear canonical wavelet space $\mathfrak{W}$ is introduced and boundedness as well as compactness of Weyl transform in Lebesgue space are discussed.

Homogeneity in mathematics: what, why and how

Christophe Rabut
Institut National des Sciences Appliquées, Toulouse, France
christophe.rabut@insa-toulouse.fr

joint work with Florian Heinrich

Looking at homogeneity of formulae is very common in physics, but very unusual in mathematics. However the strong link between physics and mathematics shows that if homogeneity is useful in physics (where the word “dimension” is often used in a bit different meaning than in the mathematical world), it should also be useful in mathematics.

We first show in this paper how to handle homogeneity in mathematical formulae, which gives the possibility of checking general and particular formulae. To do so we associate an abstract “nature” to any variable or abstract constant, or even to numbers, in a way which fulfills the “homogeneity constraints” (such as adding only quantities of same nature) in the given problem, and then we check that the result (and any intermediate computation) also fulfills the same rules. Then, in applications, these “natures” will be allowed to have any “value” connected to the application. This means for example in physical applications, a nature can take the value “length”, “time”, “strength” or so. Then, for numerical applications we can use appropriate units to each nature used (such as meters, centimeters, seconds, days...).

We give some examples where homogeneity considerations can help in remembering some formulae (useful for students!) avoiding some common errors or help solving some difficulties.

We propose a “theory of homogeneity” and define the multiplicative “group of natures”, which, when with at most 7 independent natures, is isomorph to the group of natures generated by same number of natures of the SI-system. This formalizes the possible operations between these abstract natures.

Depending on the time granted for the talk, we will show how homogeneity considerations already gave ideas on new formulae for some known functions, and then made possible to improve a condition number and to prove a new result.

Geometric texture transfer via alternative descriptors

Giuseppe Recupero
Department of Mathematics, University of Bologna, Italy
giuseppe.recupero3@unibo.it

joint work with Martin Huska, Serena Morigi

Geometric Texture Transfer, aimed to add finer details to surfaces, can be seen as a "vertex-texture mapping" where the values of the geometric texture source do not indicate alterations to pixel colors (as is common in computer graphics), but instead guide the change in vertex positions.

At this aim, we investigate and advocate the use of alternative descriptors (encodings) to the vertex Euclidean coordinates for surface representation, such as the normal-controlled coordinates [1] and the mean value encoding [2]. These representations, in general, encode the underlying geometry by describing relative position of a vertex with respect to its local neighborhood, with different levels of invariance to rigid transformation and scaling. We formulate the geometric texture transfer task as a constrained variational nonlinear optimization model that combines a fidelity term on the encoding with constraints preserving the original underlying shape of the target surface.

In contrast to other existing methods, which rely on the strong assumptions of bijectivity and equivalency in local connectivity, we only assume the patch boundaries are polygonal approximants of oriented, closed, simple curves in $\mathbb{R}^3$ with the same number of vertices and the correspondence between a few inner vertices. The proposed geometric texture transfer model is then efficiently solved by nonlinear optimization methods.

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Recognition and fitting of curves and surfaces in 3D digital models via the Hough transform technique

Chiara Romanengo
CNR - IMATI
chiara.romanengo@ge.imati.cnr.it

joint work with Silvia Biasotti, Bianca Falcidieno

Curves and surface primitives have an important role in conveying an object shape and their recognition finds significant applications in manufacturing, art, design and medical applications. In addition, when 3D models are acquired by scanning real objects, the resulting geometry does not explicitly encode these curves and primitives, especially when it is affected by noise or missing parts. The knowledge of the parts that compose a 3D model allows the reconstruction of the model itself.

The problem of recognising curves and surfaces and providing a mathematical representation of them can be addressed using the Hough transform technique (HT), which in literature is mainly used to recognise curves in the plane and planes in space. Only in the last few years it has been explored for the fitting of space curves. Such a technique is robust to noise and does not suffer from missing parts. In our approach, we take advantage of a recent HT formulation for algebraic curves to define both parametric and implicit curve representations. In this work, we will present and analyse the fit curves and surfaces in 3D digital models, both meshes and point clouds. We will also show some applications of this method to the archaeological domain and to industrial objects. The mathematical representation of curves and primitives obtained with our method allows to simplify or refine the models in an appropriate way.

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Exact sphere representations over Platonic solids based on rational multisided Bézier patches

Ada Šadl Praprotnik
Institute of Mathematics, Physics and Mechanics, Slovenia
ada.sadl-praprotnik@imfm.si

joint work with Jan Grošelj

A class of multisided Bézier patches, the so-called S-patches [3], unify triangular and tensor product Bézier patches and at the same time provide their generalization, namely an S-patch can be defined over any convex n-sided polygon, $n \geq 3$. It is obtained by first embedding the polygon into the simplex of dimension $n-1$ and then defining the multivariate rational Bézier patch of degree d over the simplex.

In this talk we use S-patches to obtain exact sphere representations over Platonic solids inscribed into the sphere. First, we present a general method, based on the properties of the Platonic solids, that utilizes the (inverse) stereographic projection and enables exact representation of a sphere section in terms of an S-patch. Then, we apply the method to the faces of all five Platonic solids inscribed into the sphere. Depending on the Platonic solid, the obtained S-patches are defined over triangular, square or pentagonal domains. This approach unifies two previously known constructions based on triangular and tensor product Bézier patches [1, 2] and introduces three new patches that together enable the representation of the sphere over all five Platonic solids.

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Best approximations of matrices and differential operators

Espen Sande
EPFL
espen.sande@epfl.ch

joint work with Michael Floater, Carla Manni, Hendrik Speleers

It is well known that if the singular values of a matrix are distinct, then its best rank-n approximation in the Frobenius norm is uniquely determined and given by the truncated singular value decomposition. On the other hand, this uniqueness is in general not true for best rank-n approximations in the spectral norm. In this talk we relate the problem of finding best rank-n approximations in the spectral norm to Kolmogorov n-widths and corresponding optimal spaces. By providing new criteria for optimality of subspaces with respect to the n-width, we describe a large family of best rank-n approximations to a given matrix. This results in a variety of solutions to the best low-rank approximation problem and provides alternatives to the truncated singular value decomposition. This variety can be exploited to obtain best low-rank approximations with problem-oriented properties.

We further discuss the generalization of these results to compact operators in L^2, and explain how they can be used to both describe the out-performance of smooth spline approximations of solutions to differential equations when compared to classical finite element methods, and to solve the outlier-problem in isogeometric analysis.

Physics informed neural network for spline approximations

Vincenzo Schiano Di Cola
Università degli Studi di Napoli Federico II
vincenzo.schianodicola@unina.it

joint work with Salvatore Cuomo

Physics-informed neural networks (PINNs) can be trained with little to no ground truth data and learn continuous solutions to PDEs. While most PINN algorithms available in the literature minimize the residual of the governing partial differential equation, other approaches have been proposed, such as minimizing the system's variational energy [1]. Geometry-aware trial functions in artificial neural networks have also been used to improve deep learning training for partial differential equations.Fast and accurate numerical schemes for finding approximate solutions are very appealing for applications such as physics engines in computer games and computer generated imagery (CGI). There are already examples in the literature of using a Hermite spline CNN to obtain continuous solutions that can be trained using only a physics informed loss [2]. We show the benefits and drawbacks of such PINN-based techniques for Splines in approximating curves.

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High-order numerical integration for trimmed isogeometric analysis

Felix Scholz
Johannes Kepler University Linz
felix.scholz@jku.at

joint work with Bert Jüttler

The numerical integration on domains that are cut by an implicitly defined boundary curve or surface is an important problem that arises when solving partial differential equations on trimmed computational domains using isogeometric analysis. Since the assembly of the system matrices can be a bottleneck for the overall computation, efficient methods for the numerical integration are needed. At the same time, in order to guarantee optimal convergence orders, the method for numerical integration needs to achieve an approximation order that is equivalent to the approximation order of the spline spaced used for solving the PDE.

We present a method for the numerical integration on trimmed computational domains that is based on a local error correction approach. In each cut integration cell, we first find a linear approximation to the trimming boundary. Then, we increase the approximation order by adding correction terms based on a Taylor expansion. The computational complexity of the resulting method is equivalent to the complexity of a standard element-wise Gaussian quadrature on a non-trimmed domain.

The first order correction term results in a fourth-order convergence rate in each cut integration cell and is therefore suited for isogeometric analysis with quadratic splines. With the help of a general transport theorem for moving domains defined by implicitly defined curves, all higher-order correction terms can be computed [1]. This makes it possible to obtain high-order quadrature rules without sacrificing the computational complexity. The method can therefore be used for high-order isogeometric analysis.

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A linear algebra approach to rational PH Curves

Hans-Peter Schröcker
University of Innsbruck
hans-peter.schroecker@uibk.ac.at

joint work with Zbyněk Šír

In [2] the authors have presented a novel approach to rational PH curves. Instead of computing them as envelope of a rational family of osculating planes [1], spherical tangent indicatrix and denominator polynomial are prescribed. Computing the numerator polynomial of a given degree than amounts to solving a system of linear equations. Its solution PH curves form a vector space $\mathcal{R}$.

The system of equations is not only modestly sized but also bears a lot of structure that allows for a more detailed analysis. It is, for example, possible to compute a canonical basis of $\mathcal{R}$ in terms of a partial fraction decomposition and to just extend this basis in case more degrees of freedom are desired.

The main purpose of basis constructions is applications to interpolation problems involving rational PH curves. Linear interpolation conditions are easily included but, in a suitable formulation, it is also possible to satisfy linear inequality constraints that govern zeros of the numerator polynomial. This helps to avoid cusps of the interpolant – something that is not easily possible with envelope approaches.

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Splines on curved triangulations and applications

Larry L. Schumaker
Vanderbilt University
larry.schumaker@gmail.com

Many practical problems involve finding an approximation to a function defined on a planar domain Ω with a curved boundary. We discuss a new approach to such problems which involves working with curved triangulations of Ω, and with polynomial splines defined on such triangulations. First we discuss an algorithm for constructing curved triangulations. Then we show how to store and evaluate polynomial splines on such triangulations. Next we construct a quasi-interpolant which leads to optimal order error bounds. We then discuss how to use the splines for interpolation and for data fitting. Finally, we also discuss methods based on such splines for the numerical solution of elliptic boundary value problems. These methods have several significant advantages over existing methods, including IGA.

On rate of convergence of matrix means of corrected Fourier series

Uaday Singh
Department of Mathematics
Indian Institute of Technology, Roorkee, India
uaday.singh@ma.iitr.ac.in

joint work with Birendra Singh

The Gibbs phenomenon is a key obstacle in many practical applications of Fourier series. The corrected Fourier series combines Fourier series with polynomials to control Gibbs phenomenon for functions having finite number of jump discontinuities and extremas. In this paper, we estimate the rate of convergence of matrix means of the corrected Fourier series followed by some corollaries and examples.

Layer potentials near surfaces with spherical topology

Chiara Sorgentone
University of Rome "La Sapienza"
`chiara.sorgentone@uniroma1.it`

joint work with Anna-Karin Tornberg

The quadrature error associated with a regular quadrature rule for evaluation of a layer potential increases rapidly when the evaluation point approaches the surface and the integral becomes nearly singular, and a specialized quadrature method must be used to keep errors low. The increased accuracy that these special methods can provide comes at an additional computational cost, and it is therefore desirable to have error estimates that can be used to determine when the accuracy will be insufficient and a special quadrature method must be applied. We study such quadrature error estimates for the Gauss-Legendre and the trapezoidal rules, when applied to evaluate layer potentials defined over smooth surfaces with spherical topology. The estimates have no unknown coefficients and can be efficiently evaluated given the discretization of the surface, invoking a local one-dimensional root-finding procedure. They are derived starting with integrals over curves, using complex analysis involving contour integrals, residue calculus and branch cuts [1]. The error estimates derived have a sufficiently low computational cost, to be practically useful. Numerical examples will be shown to illustrate the performance of the quadrature error estimates.

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Almost-C^1 splines

Deepesh Toshniwal
Delft Institute of Applied Mathematics, Delft University of Technology
d.toshniwal@tudelft.nl

joint work with Thomas Takacs

Isogeometric Analysis [1] generalizes classical finite element analysis and, at the same time, intends to seamlessly unify it with the field of Computer-Aided Design. Achieving this latter objective would encapsulate the entire engineering design-through-analysis workflow in a uniform framework, yielding a significant boost to the efficiency of current engineering workflows. A central problem in achieving this objective is design and analysis of complex two and three dimensional geometries of arbitrary topologies. This requires moving beyond splines on structured quadrilateral and hexahedral meshes – globally structured meshes cannot be used to represent arbitrary geometries and parameterization singularities (i.e., extraordinary points, polar points and extraordinary edges) must be introduced. Thus, the design and analysis of complex geometries requires that we construct and study spaces of smooth splines on unstructured meshes. This talk will present an overview of a recently proposed analysis-suitable spline constructions called Almost-C^1 splines [2].

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Multi-sided spline interpolation of curve networks

Márton Vaitkus
Budapest University of Technology and Economics
Department of Control Engineering and Information Technology
vaitkus@iit.bme.hu

joint work with Péter Salvi, Tamás Várady

Bézier and B-spline/NURBS surfaces are widely used in computer-aided geometric design, as well as isogeometric analysis. The 4-sided structure of tensor-product patches is considered the main limitation of the standard NURBS technology, and finding compatible surface representations over non-rectangular domains is a topic of intense research.

We give an overview of recent generalizations of Hermite/Coons [1], Bézier [2] and B-spline [3] surfaces that combine re-parameterized boundary interpolants ("ribbons") over multi-sided domains, and discuss a new procedural approach to curve network interpolation.

This work is supported by the Hungarian Scientific Research Fund (OTKA, No. 124727).

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Geometric approximation of the sphere by biquadratic tensor polynomial spline patches

Aleš Vavpetič
University of Ljubljana
ales.vavpetic@fmf.uni-lj.si

joint work with Emil Žagar

We study the optimal approximation of spherical squares by parametric biquadratic tensor patches if the measure of quality is the radial error. As a consequence, optimal approximations of the unit sphere by biquadratic tensor parametric polynomial spline patches underlying on cube inscribed in the unit sphere are provided.

Optimized dual interpolating subdivision schemes

Alberto Viscardi
Università di Torino
alberto.viscardi@unito.it

This work focus on the non-stepwise interpolation property of the recently introduced dual interpolating schemes. New differences between schemes having an odd or an even dilation factor are highlighted. Dual interpolating schemes with an odd dilation factor are proven to satisfy a 2-step interpolation property, while an even dilation factor corresponds to a completely non-stepwise interpolating process. These facts are then exploited in order to optimize the implementation of dual interpolating schemes. The simple idea is to compensate the natural loss of memory of a dual interpolating scheme by storing, at each subdivision level, all the computed points that would be part of the resulting limit curve.

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Source camera identification through noise information

Domenico Vitulano
University of Rome, "La Sapienza"
domenico.vitulano@uniroma1.it

joint work with Vittoria Bruni, Silvia Marconi

Source camera identification is one of the most challenging problems in forensics. It is mainly based on detecting the digital fingerprint of a generic device during the acquisition phase by just exploiting the acquired images — or videos. This task is still an open problem because of the difficulties in effectively recovering it from a generic image. It is well-known that camera sensor plays a key role because of its intrinsic and unique peculiarities, such as pixel defects, sensor dust, Fixed Pattern Noise (dark currents) (FPN) and Photo Response Non-Uniformity (PRNU) noise, that can be used for individual source identification. This talk will focus on some mathematical properties of PRNU, mainly oriented to be discriminative as well as robust to camera identification. Specifically, the properties of some well-known transforms will be investigated in order to assess their discriminative role. The first approach involves a study on Fourier spectrum decay whenever applied on a monotonic rearrangement of Flat Fields — a version of PRNU acquired in ideal conditions. It will be shown that spectrum decay embedded in some machine learning tools (Support Vector Machine) can allow a correct classification, in terms of brand and model, with high accuracy. The second approach is instead oriented to investigate the potential of Radon transform applied on a rearranged version of Flat Fields. In particular, the spatial rearrangement highlights some typical geometrical structures that clearly identify the single brand as well as the specific device. These geometrical structures are described by the distribution of the values of the Radon transform that, by definition, is sensitive to the linear image components. Preliminary results achieved on some flat field images show that the proposed PRNU feature allows for the detection of the image source device with high accuracy by using standard distances for probability density functions.

Interpolation of planar G^1 data by Pythagorean-hodograph cubic biarcs with prescribed arc lengths

Emil Žagar
University of Ljubljana
emil.zagar@fmf.uni-lj.si

joint work with Grega Cigler

The interpolation of two points and two tangent directions by planar parametric cubic curves with prescribed arc lengths will be considered. It is well known that this problem is highly nonlinear if standard cubic curves are used. However, if Pythagorean-hodograph (PH) curves are considered, the problem simplifies due to their distinguished property that the arc length is a polynomial function of its coefficients. Since a single segment of a PH cubic curve does not provide enough free parameters, the so called PH cubic biarcs will be used. The lookup table of the solutions will be given enabling an easy implementation of the described method. Some quantities arising from geometric properties of the resulting curves will suggest the most appropriate one. Several numerical examples will be given together with an example of approximation of an analytic curve by G^1 PH cubic biarc spline curve. Finally, numerical estimation of the approximation order will be presented.

Contributed posters

Triangular spline quasi-interpolation and its application in terrain modelling

Domingo Barrera
University of Granada
dbarrera@ugr.es

joint work with Salah Eddargani, María José Ibáñez, Juan Francisco Reinoso, Sara Remogna

In this work we propose the construction of a C^2 triangular spline quasi-interpolant over the type-1 triangulation of the real plane spanned by the directions $e_1 = (h, 0)$, $e_2 = (0, h)$ and $e_3 = e_1 + e_2$, $h > 0$. Instead of expressing the quasi-interpolant Qf of a given function f in terms of a basis of B-splines, it is constructed by directly setting the coefficients of the Bernstein-Bézier representation of the restriction of Qf to each macro-triangle. Each coefficient will be determined from the values of f at the points in a neighbourhood of the macro-triangle under consideration, making use of rules that will guarantee the required regularity and order of approximation.

The resulting scheme is adapted to deal with functions defined on a rectangular domain, and then applied to approximate a Digital Elevation Model.

A general class of super-convergent quasi-interpolation splines and their applications

Salah Eddargani
University of Granada
seddargani@correo.ugr.es

joint work with Domingo Barrera, María José Ibáñez

In this work, we focus on a class of functions, that are contained in the null space of certain second-order differential operators with constant coefficients. We refer to this class of functions as **D**-polynomials. The splines derived from these **D**-polynomials contain strictly the polynomial, hyperbolic and trigonometric splines. This work aims to provide a normalized B-spline-like representation for a general class of splines produced from **D**-polynomials. A geometric approach to the construction of locally supported non-negative B-spline-like basis functions will be presented. Moreover, and with the aid of control polynomial theory, a family of super-convergent quasi-interpolation schemes will be provided. These schemes can be useful in approximating a Digital Elevation Model.

Downscaling a digital elevation model from quasi-interpolation

María José Ibáñez
University of Granada
mibanez@ugr.es

joint work with Francisco Javier Ariza, Domingo Barrera, Salah Eddargani, Juan Francisco Reinoso

Many geospatial processes need to resample a Digital Elevation Model and consolidated resampling algorithms are used. However, new algorithms can still be explored, particularly those based on splines with C^2 continuity, that will allow variables involving derivatives, such as curvature and slope, to be calculated more naturally. In this work, such an algorithm is proposed as well as a combination of new and traditional ways of evaluating the accuracy achieved with this algorithm in the resampling process. A visual evaluation based on thresholds is proposed for the altimetric study, while horizontal accuracy can be addressed by techniques based on particle image velocimetry.

Conversion between CAD models and blending spline surfaces

Tatiana Kravetc
UiT – The Arctic University of Norway
tatiana.kravetc@uit.no

The proposed method demonstrates the conversion between a NURBS CAD model and a blending type spline surface with the purpose of exploiting special combined expo-rational blending spline basis functions in the isogeometric analysis [1].

The usage of blending spline construction in the isogeometric framework mixes standard finite element and NURBS-based approaches. Strict locality and smoothness are the main properties of the blending splines that make them beneficial in the isogeometric context.

The presented scheme extends the approach considered in [2]. The transformation between a NURBS patch and a blending spline surface element is achieved using a linear extraction operator that maps the local combined expo-rational basis functions to the global B-spline basis and vice versa.

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p-Laplacian based geometric deep learning for mesh processing

Damiana Lazzaro
Department of Mathematics , University of Bologna, Italy
damiana.lazzaro@unibo.it

joint work with Serena Morigi, Silvia Tozza

Geometric deep learning aims to extend standard deep learning technologies to geometric data such as graphs and manifold data, see [1]. We consider a mesh to be a discretization of a continuous 2-manifold embedded in $\mathbb{R}^3$, and we introduce a new graph convolutional mechanism which is derived from the solution of a variational approach to a p-Laplacian regularization problem. This allows us to design a Graph Neural Network architecture where node features together with positional encodings derived from the mesh topology are jointly processed to produce simultaneously feature learning and topology evolution.

The final goal of this work is to apply our data-driven approach in the context of geometry processing. For example, mitigating the noise of acquired meshes (mesh denoising) or reducing the number of representative geometric elements of a mesh (mesh decimation), while preserving the original topology and a good approximation to the original geometry.

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A reverse non-stationary mixed trigonometric and hyperbolic B-splines subdivision scheme

Mohammed Oraiche
Hassan First, University of Settat
Faculté des Sciences et Technique, Laboratory MISI, Morocco.
mohammed.oraiche@gmail.com

joint work with Abdellah Lamnii, Mohamed Louzar, Mohamed-Yassir Nour and Ahmed Zidna.

In this paper, two new families of non-stationary subdivision schemes are introduced. The schemes are constructed from uniform mixed trigonometric and hyperbolic B-splines with multiple knots of orders 3 and 4, respectively. Then, we construct a third-order reverse subdivision framework. For that, we derive a mixed trigonometric an hyperbolic multi-resolution mask based on their third-order subdivision filters. Numerical examples are given to show the performance of the new schemes in reproducing different shapes of initial control polygons.

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In Memoriam Paul Sablonnière

Sara Remogna
Department of Mathematics, University of Torino, Italy
sara.remogna@unito.it

joint work with Francesca Pelosi, Alessandra Sestini

On the 29th of March 2022 Paul Sablonnière passed away. *Professor Paul*, as he was friendly called by his colleagues , was not only a brilliant mathematician but also a great man. But most of all he was a wonderful teacher, as one of his mission has always been to encourage and to instill enthusiasm and passion for the knowledge to his collaborators and his students.

With this contribution we aim to remember him, his research, the many fruitful collaborations he had, his kind and elegant character.

In Memoriam Maria Charina

Lucia Romani
Department of Mathematics, University of Bologna, Italy
lucia.romani@unibo.it

joint work with Costanza Conti, Mariantonia Cotronei

This wants to be a contribution from Maria's closest collaborators and friends to remember her research interests and her main results in subdivision, frames, wavelets and related applications.

List of Participants

1. **Mohamed Ajeddar**
Hassan First University, Settat, Morocco
m.ajeddar@uhp.ac.ma

2. **Francesc Aràndiga**
Universitat de València, Spain
arandiga@uv.es

3. **Antonio Bacciaglia**
University of Bologna, Italy
antonio.bacciaglia2@unibo.it

4. **Domingo Barrera**
University of Granada, Spain
dbarrera@ugr.es

5. **Shubhashree Bebarta**
Veer Surendra Sai University of Technology,
Burla, Sambalpur, Odisha, India
shubhashreebebarta2009@gmail.com

6. **Carolina Beccari**
University of Bologna, Italy
carolina.beccari2@unibo.it

7. **Cesare Bracco**
University of Florence, Italy
cesare.bracco@unifi.it

8. **Francesco Calabrò**
Dipartimento di Matematica ed Applicazioni,
Università degli Studi di Napoli "Federico II", Italy
calabro@unina.it

9. **Simone Cammarasana**
CNR IMATI, Italy
simone.cammarasana@ge.imati.cnr.it

10. **Rosanna Campagna**
University of Campania "Luigi Vanvitelli", Italy
rosanna.campagna@unicampania.it

11. Filip Chudy
University of Wroclaw, Poland
`fch@cs.uni.wroc.pl`

12. Costanza Conti
Università di Firenze, Italy
`costanza.conti@unifi.it`

13. Mariantonia Cotronei
Università Mediterranea di Reggio Calabria, Italy
`mariantonia.cotronei@unirc.it`

14. Salvatore Cuomo
Unversity of Naples "Federico II", Italy
`salvatore.cuomo@unina.it`

15. Bruno Degli Esposti
University of Florence, Italy
`bruno.degliesposti@unifi.it`

16. Francesco Dell'Accio
Department of Mathematics and Computer Science
University of Calabria, Italy
`francesco.dellaccio@unical.it`

17. Giuseppe Alessio D'Inverno
Università degli Studi di Siena, Italy
`giuseppealessio.d@student.unisi.it`

18. Rosa Donat
Universitat de Valéncia, Spain
`donat@uv.es`

19. Salah Eddargani
University of Granada, Spain
`seddargani@correo.ugr.es`

20. Karla Ferjančič
University of Primorska, FAMNIT and IAM, Slovenia
`karla.ferjancic@iam.upr.si`

21. Frank Filbir
Helmholtz Center Munich & Technische Universität
München, Germany
`filbir@helmholtz-muenchen.de`

22. Carlo Garoni
Università di Roma “Tor Vergata”, Italy
garoni@mat.uniroma2.it

23. Carlotta Giannelli
University of Florence, Italy
carlotta.giannelli@unifi.it

24. Jan Grošelj
University of Ljubljana, Slovenia
jan.groselj@fmf.uni-lj.si

25. Martin Huska
University of Bologna, Italy
martin.huska@unibo.it

26. María José Ibáñez
Universitat de Granada, Spain
mibanez@ugr.es

27. Sofia Imperatore
University of Florence, Italy
sofia.imperatore@unifi.it

28. Ioannis Ivrissimtzis
Department of Computer Science
University of Durham
ioannis.ivrissimtzis@durham.ac.uk

29. Tadej Kanduč
University of Ljubljana, Slovenia
tadej.kanduc@fmf.uni-lj.si

30. Marjeta Knez
Faculty of Mathematics and Physics
University of Ljubljana, Slovenia
marjetka.knez@fmf.uni-lj.si

31. Alexander Komar
Institute of Computer Graphics and Knowledge Visualization
Graz University of Technology, Austria
a.komar@cgv.tugraz.at

32. Jiří Kosinka
University of Groningen, Netherlands
j.kosinka@rug.nl

33. Aljaž Kosmač
University of Primorska, Slovenia
`aljaz.kosmac@iam.upr.si`

34. Tatiana Kravetc
UiT – The Arctic University of Norway, Norway
`tatiana.kravetc@uit.no`

35. Damiana Lazzaro
University of Bologna, Italy
`damiana.lazzaro@unibo.it`

36. Dany Leviatan
Tel Aviv University, Israel
`leviatan@tauex.tau.ac.il`

37. Tom Lyche
University of Oslo, Norway
`tom@math.uio.no`

38. Carla Manni
University of Rome "Tor Vergata", Italy
`manni@mat.uniroma2.it`

39. Michelangelo Marsala
Inria Sophia Antipolis-Méditerranée
Université Côte d'Azur, France
`michelangelo.marsala@inria.fr`

40. Wael Mattar
Tel Aviv University, Israel
`waelmattar.95@gmail.com`

41. Mariarosa Mazza
University of Insubria, Italy
`mariarosa.mazza@uninsubria.it`

42. Serena Morigi
University of Bologna, Italy
`serena.morigi@unibo.it`

43. Mohamed-Yassir Nour
Hassan First University of Morocco,
University of Lorraine, France
`mohamedyassir.nour@gmail.com`

44. Mohammed Oraiche
Hassan First University of Settat, Maroc
mohammed.oraiche@gmail.com

45. Francesco Patrizi
Max Planck Institute for Plasma Physics (IPP), Germany
francesco.patrizi@ipp.mpg.de

46. Enza Pellegrino
DIIIE, Italy
enza.pellegrino@univaq.it

47. Francesca Pelosi
Università di Roma "Tor Vergata", Italy
pelosi@mat.uniroma2.it

48. Emma Perracchione
Politecnico di Torino, Italy
emma.perracchione@polito.it

49. Laura Pezza
S.B.A.I., Università di Roma "La Sapienza", Italy
laura.pezza@uniroma1.it

50. Francesca Pitolli
Università di Roma "La Sapienza", Italy
francesca.pitolli@uniroma1.it

51. Akhilesh Prasad
Indian Institute of Technology (Indian School of Mines)
Dhanbad, India
aprasad@iitism.ac.in

52. Hartmut Prautzsch
Karlsruher Institut für Technologie, Germany
prautzsch@kit.edu

53. Christophe Rabut
INSA Toulouse, France
christophe.rabut@insa-toulouse.fr

54. Giuseppe Antonio Recupero
University of Bologna, Italy
giuseppe.recupero3@unibo.it

55. Sara Remogna
Università degli Studi di Torino, Italy
sara.remogna@unito.it

56. Chiara Romanengo
CNR-IMATI, Italy
chiara.romanengo@ge.imati.cnr.it

57. Lucia Romani
University of Bologna, Italy
lucia.romani@unibo.it

58. Ada Šadl Praprotnik
Institute of Mathematics, Physics and Mechanics
University of Ljubljana, Slovenia
ada.sadl-praprotnik@imfm.si

59. Svajñas Sajavičius
Kaunas University of Technology, Lithuania
svajunas.sajavicius@ktu.lt

60. Maria Lucia Sampoli
Department of Information Engineering and Mathematics,
University of Siena, Italy
marialucia.sampoli@unisi.it

61. Espen Sande
EPFL, Switzerland
espen.sande@epfl.ch

62. Vincenzo Schiano Di Cola
University of Naples Federico II, Italy
vincenzo.schianodicola@unina.it

63. Felix Scholz
Johannes Kepler University Linz, Austria
felix.scholz@jku.at

64. Carola-Bibiane Schönlieb
Centre for Mathematical Sciences, Cambridge, United Kingdom
cbs31@cam.ac.uk

65. Hans-Peter Schröcker
University of Innsbruck, Austria
hans-peter.schroecker@uibk.ac.at

66. Larry L. Schumaker
Vanderbilt University, USA
larry.schumaker@gmail.com

67. Ouakkas Seddik
University of Saïda, Algérie
souakkas@yahoo.fr

68. Alessandra Sestini
University of Firenze, Italy
alessandra.sestini@unifi.it

69. Somveer Singh
Indian Institute of Technology BHU Varanasi, India
rathaurbhu.90@gmail.com

70. Uaday Singh
Department of Mathematics, Indian Institute of Technology
Roorkee, India
uadayfma@iitr.ac.in

71. Chemikh Smail
Faculty of Mathematics U.S.T.H.B Alger, Algèrie
sm.chemikh@gmail.com

72. Chiara Sorgentone
Università di Roma "La Sapienza", Italy
chiara.sorgentone@uniroma1.it

73. Hendrik Speleers
University of Rome "Tor Vergata", Italy
speleers@mat.uniroma2.it

74. Thomas Takacs
Johann Radon Institute for Computational and
Applied Mathematics (RICAM), Austria
thomas.takacs@ricam.oeaw.ac.at

75. Deepesh Toshniwal
Delft Institute of Applied Mathematics,
Delft University of Technology, Netherlands
d.toshniwal@tudelft.nl

76. Silvia Tozza
Department of Mathematics
University of Bologna, Italy
silvia.tozza@unibo.it

77. Michael Unser
EPFL, Switzerland
michael.unser@epfl.ch

78. Márton Vaitkus
Budapest University of Technology and Economics, Hungary
vaitkus@iit.bme.hu

79. Aleš Vavpetič
Faculty of Mathematics and Physics
University of Ljubljana, Slovenia
ales.vavpetic@fmf.uni-lj.si

80. Alberto Viscardi
Dipartimento di Matematica "G. Peano"
Università di Torino, Italy
alberto.viscardi@unito.it

81. Domenico Vitulano
University of Rome "La Sapienza", Italy
domenico.vitulano@uniroma1.it

82. Johannes Wallner
TU Graz, Austria
j.wallner@tugraz.at

83. Emil Žagar
University of Ljubljana
Faculty of Mathematics and Physics, Slovenia
emil.zagar@fmf.uni-lj.si

84. Zeze Zhang
Univerisity of Alberta, Canada
zeze@ualberta.ca

85. Zvi Ziegler
Technion, Haifa, Israel
ziegler@technion.ac.il