Abstract

In this talk, I will show how to construct some solutions for the fractional Yamabe problem which are singular along a prescribed set $\Sigma$. The well known Yamabe problem appears when we try to find metrics which are conformal to the one given in a manifold and which have constant scalar curvature. The fractional Yamabe problem with singularities arises in conformal geometry when we try to find metrics that are conformal to the given one, have constant fractional curvature and are singular along the prescribed set $\Sigma$. It is equivalent to finding positive and smooth solutions for

$$(-\Delta)^{\gamma} u = c_{n,\gamma} u^{\frac{n+2\gamma}{n-2\gamma}}, u > 0 \text{ in } \mathbb{R}^n \setminus \Sigma.$$ 

The fractional curvature, a generalization of the classical scalar curvature, is defined from the conformal fractional Laplacian, which is a non-local operator constructed on the conformal infinity of a conformally compact Einstein manifold. This “new” non-local curvature provides us with a generalization of some known local curvatures (scalar curvature, $Q$-curvature, mean curvature...) to the non-local setting. To solve the problem, we develop some new techniques: some new tools for fractional order ODEs and non-local gluing will be shown during the talk. This time, I will focus on the case of isolated singularities and I will give some ideas about how to use this result for the case of a higher-dimension smooth submanifold.

This study is divided into different works in which I have collaborated with Weiwei Ao, Hardy Chan, Marco Fontelos, Manuel del Pino, María del Mar González and Juncheng Wei.