A Brezis-Nirenberg type result for mixed local and nonlocal operators

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In this seminar we present some existence results, in the spirit of the celebrated paper by Brezis and Nirenberg (CPAM, 1983), for a *critical problem* driven by a mixed local and nonlocal linear operator. More precisely, given a bounded open set $\Omega \subseteq \mathbb{R}^n$ (with $n \geq 4$) and $\lambda > 0$, we consider the *perturbed critical problem*

$$
\text{(P)} \qquad
\begin{cases}
-\Delta u + (-\Delta)^s u = u^{\frac{n+2}{n-2}} + \lambda u^p & \text{in } \Omega \subseteq \mathbb{R}^n, \\
u \gneq 0 & \text{in } \Omega, \\
u = 0 & \text{in } \partial\Omega
\end{cases}
$$

and we develop an existence theory, both in the case of linear (that is, $p = 1$) and superlinear (that is, $p > 1$) perturbations. In the particular case $p = 1$, we also investigate the mixed Sobolev inequality associated with (P), detecting the optimal constant, which we show that is never achieved.

The results discussed in this talk are obtained in collaboration with S. Dipierro, E. Valdinoci and E. Vecchi.