# HOROSPHERES IN SEVERAL COMPLEX VARIABLES

LEANDRO AROSIO

Abstract. A horocycle in the the unit disk of the complex plane is a euclidean disk which is internally tangent to a point p of the boundary of the disk. Horocycles are limits of Poincaré balls as the center moves towards the point p while the radius grows suitably. The classical Julia lemma is a boundary version of the Schwarz lemma and shows that horocycles are useful to understand the behaviour of a holomorphic self-map of the disk near a point of the boundary. In this talk we deal with the generalization of this concept to several complex variables: horospheres. The existence of horospheres in bounded strongly convex domains of $\mathbb{C}^n$ was proved by Abate in 1988 using Lempert's theory of complex geodesics, but it is difficult to generalize such proof to bounded strongly pseudoconvex domains, which is the natural class of domains to study in this context.

In this talk I will show how to obtain this generalization following a different route, that is, proving that the horofunction compactification of the domain is topologically equivalent to its Gromov compactification. This is a joint work with Matteo Fiacchi, Sébastien Gontard, and Lorenzo Guerini.

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# Wavelet phase retrieval for bandlimited functions

joint work with Rima Alaifari and Matthias Wellershoff

The term phase retrieval indicates a large class of problems in which one seeks to recover a signal from phaseless measurements. In particular, wavelet phase retrieval consists of recovering a signal from the magnitudes of its wavelet coefficients. This problem has already been investigated in [2] for the Cauchy wavelet transform, and later in [1] in a sign retrieval setting, i.e. both the signal and the wavelet are assumed to be real-valued. We consider real-valued bandlimited functions and we derive the first uniqueness results for the recovery of real-valued signals from wavelet magnitude measurements in the case that the wavelet can be complex-valued, i.e. phase and not merely sign information is lost in the acquisition process. In particular, we prove a first uniqueness result for wavelet phase retrieval from samples when the wavelet coefficients are complex-valued.

[1] R. Alaifari, I. Daubechies, P. Grohs, and G. Thakur. Reconstructing real-valued functions from unsigned coefficients with respect to wavelet and other frames. *Journal of Fourier Analysis and Applications*, 23(6):1480–1494, 2017.

[2] S. Mallat and I. Waldspurger. Phase retrieval for the Cauchy wavelet transform. *Journal of Fourier Analysis and Applications*, 21(6):1251–1309, 2015.

## Boundedness of Translation operator in de Branges spaces

**Carlo Bellavita**

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In this presentation, I shall talk about the vertical and the horizontal translation operators in de Branges spaces. Starting from the Plancherel-Polya inequality in Paley Wiener spaces and keeping in mind the results on the Bernstein inequality in Model spaces and in de Branges spaces, I will provide necessary and/or sufficient conditions for the boundedness of the translation operators.

These results are part of my PhD research work.

# SCHRÖDINGER OPERATORS ON LIE GROUPS WITH PURELY DISCRETE SPECTRUM

TOMMASO BRUNO

Abstract. On a Lie group $G$, let $\mathcal{L}$ be a left-invariant sub-Laplacian and $V$ be a locally integrable potential which is bounded from below. I will discuss necessary and sufficient conditions for which the spectrum of the Schrödinger operator $\mathcal{L} + V$ is purely discrete, both when $V$ is an arbitrary potential, and in the particular case when it is a polynomial on $G$ or belongs to some local Muckenhoupt class. The talk will be based on joint work with M. Calzi [1].

## References

[1] T. Bruno, M. Calzi, *Schrödinger operators on Lie groups with purely discrete spectrum*, arXiv:2108.01953

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# Carleson and Sampling Measures for Bergman Spaces on Homogeneous Siegel Domains

Given $p \in ]0, \infty[$, a Hausdorff space $X$, and a quasi-Banach space $Y$ of functions on $X$, a positive measure $\mu$ on $X$ is said to be $p$-Carleson for $Y$ if $Y \subseteq L^p(\mu)$ continuously. The measure $\mu$ is said to be $p$-sampling for $Y$ if $Y$ embeds as a closed subspace of $L^p(\mu)$.

We consider the problem of determining Carleson and sampling measures for $Y$, when $X$ is a homogeneous Siegel domain and $Y$ is a (weighted) Bergman space on $X$, that is, the space of holomorphic functions in $L^q(\nu)$, where $q \in ]0, \infty]$, and $\nu$ is a suitable 'homogeneous' measure on $X$. This is joint work with M. M. Peloso.

# WEIGHTED DYADIC HARDY INEQUALITIES

NIKOLAOS CHALMOUKIS

Abstract. The classical Hardy inequality, i.e. that the arithmetic means of a p-summable sequence are also p-summable, with its numerous variations continues to provide motivation for mathematicians. In this talk we will make a survey of the results around the two-weighted Hardy inequality on the tree of dyadic intervals, we will discuss its connections with potential theory and some of its applications in complex and harmonic analysis. Time permitting, we will also discuss some new results.

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# STRICHARTZ ESTIMATES FOR SOME VARIABLE COEFFICIENT SCHRÖDINGER OPERATORS

SERENA FEDERICO

Abstract. In this talk we will investigate the validity of Strichartz estimates for some variable coefficient Schrödinger operators. More specifically, the first part of the talk will be devoted to the validity of such estimates for some time-degenerate Schrödinger operators in the Euclidean setting. In the second part of the talk we shall change the setting to the 2-dimensional torus. Here we will show that the suitable version of Bourgain's sharp Strichartz estimate for some time-degenerate Schrödinger operators holds true. Finally, still on the 2-dimensional torus, we will consider a space-variable non degenerate class of Schrödinger operators. The corresponding local well-posedness result for the semilinear Cauchy problem in the Euclidean setting and on the 2-dimensional torus will also be given.

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# ON THE GROMOV HYPERBOLICITY OF DOMAINS IN $\mathbb{C}^n$

MATTEO FIACCHI

Abstract. In this talk we discuss the Gromov hyperbolicity of the Kobayashi distance on domains in $\mathbb{C}^n$. Gromov hyperbolicity is a weak notion of "negative curved space on large scale" that can be defined in a generic metric space and allows, among other things, to have a geodesic stability and have a natural definition of compactification, adding a "boundary at infinity". In complex analysis, this notion turns out to be very useful to prove results on holomorphic maps. We show the relationship with the concept of boundary points of finite type in the sense of D'Angelo, presenting a new result concerning the Gromov hyperbolicity of pseudoconvex finite type domains in $\mathbb{C}^2$.

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# Cassels-Montgomery lemma and almost positive kernels on Riemannian manifolds

Bianca Gariboldi

Let $(\mathcal{M}, g)$ be a $d$-dimensional compact connected Riemannian manifold. In this talk I will present the Cassels-Montogomery lemma, proving it using a kernel

$$K_X(x,y) = \sum_{m=0}^{X} h(\lambda_m/\lambda_X)\varphi_m(x)\overline{\varphi_m(y)}$$

positive up to a negligible error and such that $K_X(x,y) \approx X$, with $0 = \lambda_0^2 \le \lambda_1^2 \le \dots$ eigenvalues of the Laplace-Beltrami operator on $\mathcal{M}$ and $\{\varphi_m\}_{m=0}^{+\infty}$ an associated system of eigenfunctions. It will be shown how to build a such kernel.

# THE CAUCHY–SZEGŐ PROJECTION AND ITS COMMUTATOR FOR DOMAINS IN $\mathbb{C}^n$ WITH MINIMAL SMOOTHNESS: OPTIMAL ESTIMATES

LOREDANA LANZANI

Abstract. Let $D \subset \mathbb{C}^n$ be a bounded, strongly pseudoconvex domain whose boundary $bD$ satisfies the minimal regularity condition of class $C^2$. A 2017 result of Lanzani & Stein states that the Cauchy–Szegő projection $\mathcal{S}_\omega$ maps $L^p(bD, \omega)$ to $L^p(bD, \omega)$ continuosly for any $1 < p < \infty$ whenever the reference measure $\omega$ is a bounded, positive continuous multiple of induced Lebesgue measure. Here we show that $\mathcal{S}_\omega$ (defined with respect to any measure $\omega$ as above) satisfies explicit, optimal bounds in $L^p(bD, \Omega_p)$, for any $1 < p < \infty$ and for any $\Omega_p$ in the maximal class of $A_p$ measures, that is $\Omega_p = \psi_p\sigma$ where $\psi_p$ is a Muckenhoupt $A_p$-weight and $\sigma$ is induced Lebesgue measure. As an application, we characterize boundedness in $L^p(bD, \Omega_p)$ with explicit bounds, and compactness, of the commutator $[b, \mathcal{S}_\omega]$ for any $A_p$-measure $\Omega_p$, $1 < p < \infty$. We next introduce the notion of holomorphic Hardy spaces for $A_p$-measures, and we characterize boundedness and compactness in $L^2(bD, \Omega_2)$ of the commutator $[b, \mathcal{S}_{\Omega_2}]$ where the Cauchy–Szegő projection is defined with respect to any $A_2$ measure $\Omega_2$. Earlier results rely upon an asymptotic expansion and subsequent pointwise estimates of the Cauchy–Szegő kernel, but these are unavailable in our setting of minimal regularity of $bD$; at the same time, recent techniques that allow to handle domains with minimal regularity, are not applicable to $A_p$-measures. It turns out that the method of quantitative extrapolation is an appropriate replacement for the missing tools.

# BMO, HARDY SPACES AND CALDERÓN-ZYGMUND THEORY ON SOME NONDOUBLING TREES

MATTEO LEVI

Abstract. Calderón-Zygmund theory and the interlaced theory of Hardy and BMO spaces are well settled on spaces of homogeneous type. In recent years many attempts were made to extend the theory to the nondoubling setting. The most renowned results in this direction apply to (nondoubling) measures that fulfill some growth or some geometric condition. We consider infinite trees endowed with flow measures, which are examples of nondoubling metric measure of at least exponential growth and not satisfying the isoperimetric inequality. In this adverse setting, we develop a Calderon-Zygmund theory and we define BMO and Hardy spaces, proving a number of desired results extending the theory as known in more classical settings.

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# SHARP MULTIPLIER THEOREMS FOR GRUSHIN OPERATORS

ALESSIO MARTINI

Abstract. Grushin operators are among the simplest examples of degenerate elliptic, subelliptic operators. The existence of a Hörmander functional calculus on $L^p$ for such operators is known in great generality, but the standard techniques based on heat kernel estimates yield spectral multiplier theorems that are typically not sharp in terms of the smoothness requirement on the multiplier. In contrast, sharp results have been proved only in relatively few instances. We report on recent progress.

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# THE LEVI CORE OF A PSEUDOCONVEX DOMAIN

SAMUELE MONGODI

Abstract. In a joint paper with G. M. Dall'Ara, we introduced the notion of "core of a distribution" and applied it to the distribution of kernels of the Levi form of a CR manifold of hypersurface type. Specializing this idea to the case of a pseudoconvex domain in a complex manifold, we obtain the Levi core, which is, in a way, a distribution of subspaces of the complex tangent space with a weak integrability property. This object has proved useful in studying the Diederich-Fornaess exponent; its construction helps in obtaining coercivity estimates for the Kohn laplacian in a special class of domains and seems to be linked to the Kohn algorithm, which gives bounds for the subellipticity exponent of the $\overline{\partial}$-problem. The aim of this talk is to introduce the geometric construction of the Levi core and to give a rough sketch of the applications mentioned above.

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# EULER-MACLAURIN SUMMATION FORMULAS ON POLYHEDRA

ALESSANDRO MONGUZZI

Abstract. In this seminar I will present a multidimensional weighted Euler–MacLaurin summation formula on polyhedra as a consequence of a more general result on the series expansion of functions in multivariate Bernoulli polynomials. This is a joint work with L. Brandolini, L. Colzani, B. Gariboldi and G. Gigante.

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# REPRODUCING KERNEL FOR BERGMAN SPACES ON HOMOGENEOUS TREES

MATTEO MONTI

ABSTRACT. Bergman spaces are realized on the hyperbolic disk as holomorphic functions that are square-integrable w.r.t. weighted versions of the hyperbolic measure. Through the analogy between hyperbolic disk and homogeneous trees, Bergman spaces of harmonic functions have been introduced on homogeneous trees by J. Cohen, F. Colonna, M. Picardello and D. Singman. In collaboration with F. De Mari and M. Vallarino we show that such Bergman spaces are reproducing kernel Hilbert spaces and we provide an explicit formula for the kernels. Furthermore, we state boundedness results for the extension of the Bergman projector to weighted $L^p$-spaces.

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# THE SUBHARMONICITY INDEX OF HIGHER ORDER GRADIENT OF REGULAR FUNCTIONS

STEFANO PINTON

Abstract. It was proved by Stein and Weiss (1960) that for any harmonic function $f$ in $\mathbb{R}^n$ the power of the gradient

$$|\nabla f|^\alpha := \left(\sum_{i=1}^{n} (\partial_{x_i} f)^2\right)^{\frac{\alpha}{2}}$$

is subharmonic for $\alpha \geq \frac{n-2}{n-1}$ (when $n = 2$ the previous result means that for any $\alpha > 0$ the function $|\nabla f|^\alpha$ is subharmonic and also $\log(|\nabla f|)$ is a subharmonic function). Calderón and Zygmund (1964) extended this result to higher order gradients proving that

$$|\nabla^m f|^\alpha := \left(\sum_{|\beta|=m} |\partial^\beta f|^2\right)^{\frac{\alpha}{2}},$$

is subharmonic for $\alpha \geq \frac{n-2}{n+m-2}$ and that such lower bound is optimal. Since an holomorphic function $f : \Omega \subset \mathbb{C} \to \mathbb{C}$ is the gradient of an harmonic function, the previous results imply that if $\gamma > 0$ the function $|\nabla^m f|^\gamma$ is subharmonic for any integers $m \geq 0$ where $\nabla^0$ is meant to be the identity operator. Moreover, in this case also $\log(|\nabla^m f|)$ is subharmonic.

In the context of quaternionic, Clifford algebraic or octonionic analysis it is no more true that regular functions are the gradient of an harmonic function. Nevertheless Stein and Weiss (1968) proved that if $f$ is a monogenic quaternionic function then $|f|^\alpha$ is subharmonic for $\alpha \geq \frac{2}{3}$. Kheyfits and Tepper (2006) proved that the same result holds in the octonions for $\alpha \geq \frac{6}{7}$.

In this talk I will explain how to extend the technique of Calderón and Zygmund to higher order gradients of monogenic functions on quaternions, Clifford algebras and octonions. This talk is based on a joint work with Baracco.

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# RIESZ TRANSFORM FOR A FLOW LAPLACIAN ON HOMOGENEOUS TREES

FEDERICO SANTAGATI

We obtain weak type (1, 1) and $L^p$ boundedness, for $p \in (1, \infty)$, of the first order Riesz transform and its adjoint operator on a homogeneous tree endowed with the canonical flow measure. We will also show a negative result for the $L^\infty \to BMO$ boundedness of the Riesz transform. This is based on a joint work with Matteo Levi, Anita Tabacco and Maria Vallarino.

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# AN OVERVIEW ON THE QUATERNIONIC HARDY SPACE

GIULIA SARFATTI

Abstract. The Hardy space on the complex unit disc is a classical instance of space of holomorphic functions. Its interests are manifold: besides its centrality in complex analysis and operator theory, it is connected to the geometry of the unit disc itself. In this seminar we will review some aspects of its quaternionic counterpart, enlightening similarities and differences with the classical case.

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